Hypersonic Air Intake Design for High Performance and Starting

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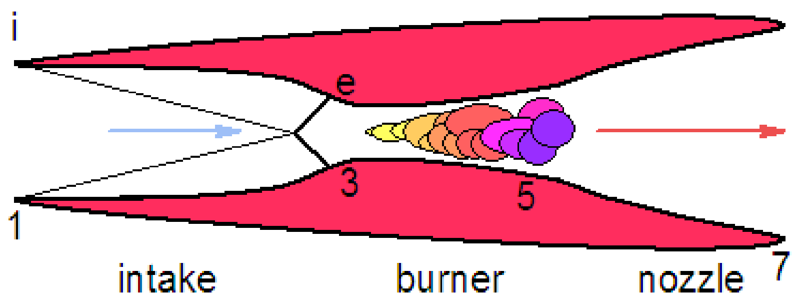


SUMMARY

Hypersonic air intake performance is defined in terms of intake capability and efficiency. The propensity for intake flow starting is measured by the Startability Index – a parameter inversely related to capability that is shown to be a meaningful and convenient measure of the startability for a hypersonic air intake. It is shown that the basic Busemann flow is adaptable to the design of high-performance air intakes. Proper choice of the strength of the terminal shock yields high-performance, modular, intake shapes, with low internal contractions, that are capable of spontaneous starting with overboard mass spillage. Some practical designs and applications are presented.

1.0 INTRODUCTION

Air-breathing aero-engines such as ramjets and scramjets, operating at supersonic and hypersonic speeds, ingest air through a converging air intake[1]. For both ramjets and scramjets at these speeds, convergence causes enough compression that no further mechanical compression is required before combustion. In a scramjet, the static temperature at combustor entry is high enough to cause the injected fuel to ignite spontaneously. The resulting lack of mechanical complexity, with practically no moving parts, puts the ramjet as well as the scramjet to overall advantage over the turbojet engine even at low Mach numbers where turbojet thermodynamic performance may exceed that of the ramjet/scramjet. For useful engine operation, at some flight conditions, the intake duct shape must be such that the required air mass flow in the duct is predictable, stable, properly conditioned (uniform in some sense) and thermodynamically efficient. In addition, the

[1] The terms *diffuser*, *inlet* and *intake* have been used to denote the leading component of an air-breathing engine. The term *diffuser* is archaic, having its origin in diverging ducts used for fans and subsonic wind tunnels. It will not be used. There is not much to choose between *intake* and *ilet*. We will use *intake* leaving *inlet* to be used occasionally to denote conditions at the entry of the *intake*.

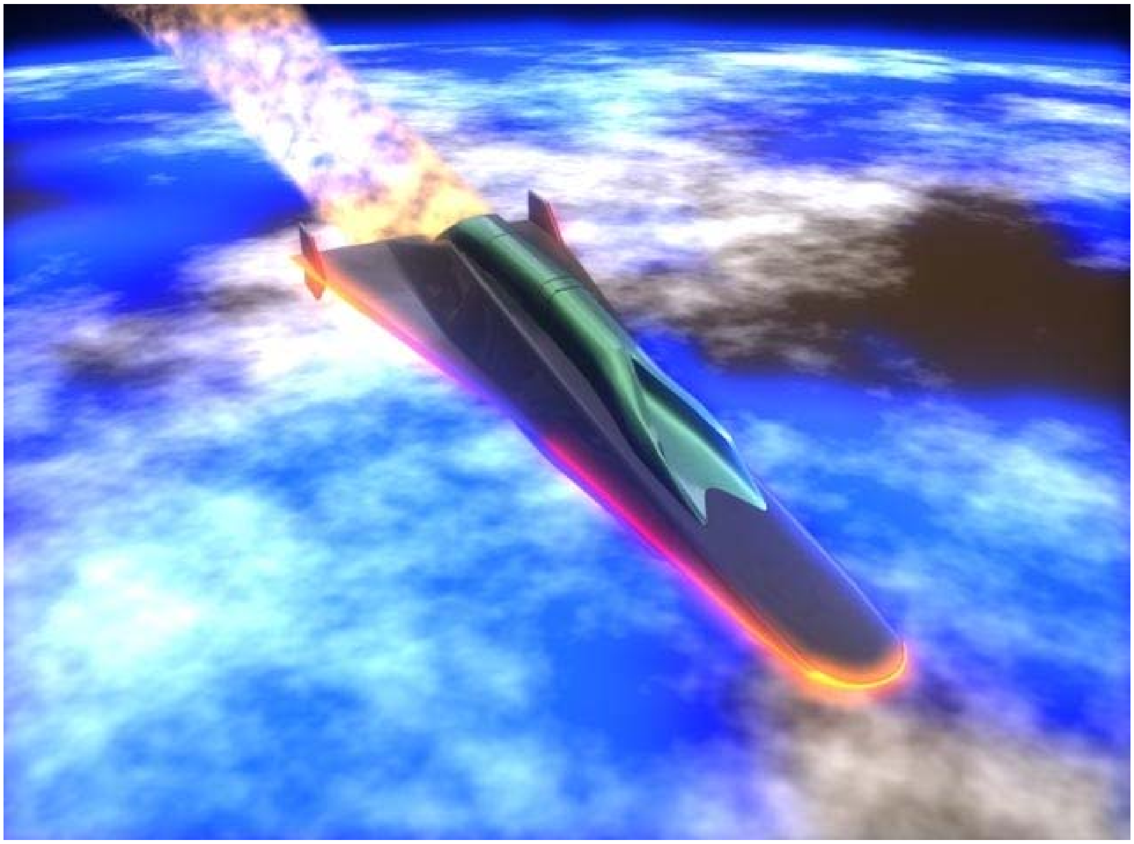
degradation of performance at off-design conditions must be minimal.[2] A comprehensive treatment of intake design considerations is given by Van Wie (2000).

The purpose of an air intake is to condition the incoming airflow for the combustor so as to obtain optimum performance from the airplane. A specific task for the intake is to lower the incoming flow Mach number to a lesser value at the combustor entry since heat addition at high Mach numbers brings with it high losses of total pressure in the propulsive stream. For a **ramjet** the flow velocity at combustor entry must be low enough not to blow out the flame and the Mach number must be low enough that heat addition in the combustor will not cause the combustor exit flow to choke. For a **scramjet** the combustor entry temperature must be high enough to maintain spontaneous combustion and, in this case, the Mach number must be high enough that heat addition will not cause choking at the combustor exit. For effective combustion, the compression ratio at a given flight altitude has to be enough to produce a combustor static pressure of at least one atmosphere. At hypersonic Mach numbers the intake efficiency must be as high as possible so as to minimize chemical non-equilibrium effects in the combustor and nozzle. All these requirements, together with the thermodynamic behavior of a supersonic flow, require **the intake flow to be converging and increasing in pressure in a contracting and compressing flow passage.**

Scramjet engine cycle calculations, for a freestream Mach number range 4 to 25, have shown that a variation of intake geometry is required to produce contraction ratios from 5 to 20 with a consequent range of compression ratios from 10 to 50. In all cases the Mach number would be reduced by a factor of about three. For optimal performance it is highly desirable to change the shape of an intake to make it stay 'on design' during a variation in flight conditions as well as to promote intake flow starting. However, the harsh thermal conditions of hypersonic flight make such changes in intake shape very difficult to implement. Considerations in this report are restricted to **fixed-geometry intakes.** For the same reasons we will not rely on wall perforations for intake flow starting.

Flow over the top surface of a hypersonic air-breathing airplane contributes little to the forces and moments. The forces acting are dominated by the engine which makes up the lower surface of the vehicle. Not only thrust but also lift, drag and moments are generated and determined largely by the engine geometry and the flow through the engine. In turn, the choice of intake geometry governs the shape of the engine as well as the airplane and the forces acting on it. Considerations of structural and thermal loads follow the logical design steps as determined by the inter-dependence of airframe and engine aerodynamics. Thus, the engine plays a major role in shaping the airplane and the **intake, in turn, has a major effect on the design of the engine**.

[2] Both ramjets and scramjets are incapable of producing thrust at zero forward speed so they need to be boosted to their start-up speeds by a turbojet or rocket propulsor. A ramjet can be started at a high subsonic speed; a scramjet combustor ignites above Mach 4. At this Mach number the scramjet's performance is better than that of the ramjet. Ramjet performance falls to uselessness above Mach 7. Scramjet performance falls to that of rockets above Mach 20 or so.



Two types of flow dimensionality, planar and axial, have been considered for scramjet intake design [Dissel et al]: a) a design based on planar shocks and Prandtl-Meyer flow and b) a design based on internal axial flow, [Keirsey, Billig (1965); [Molder, Szpiro (1966); Molder, Romeskie (1967); VanWie, Molder (2000), Ogawa et al].

In comparison with the planar flow design, the axial-flow-based intake has a smaller exposed surface area, leading to smaller heat transfer loads and smaller boundary layer losses with a consequent advantage in engine propulsion performance [VanWie (2000)].

a) design based on planar flow
[Dissel et al]

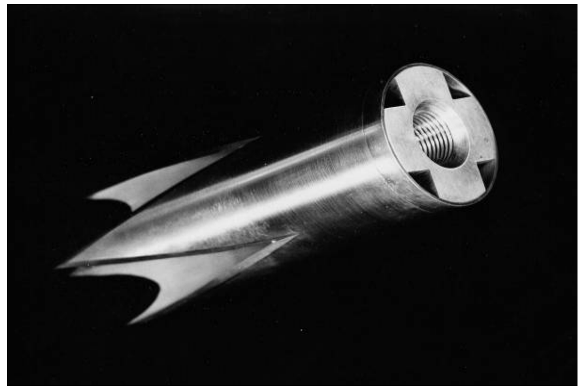
b) design based on axial flow
[Molder & Romeskie]

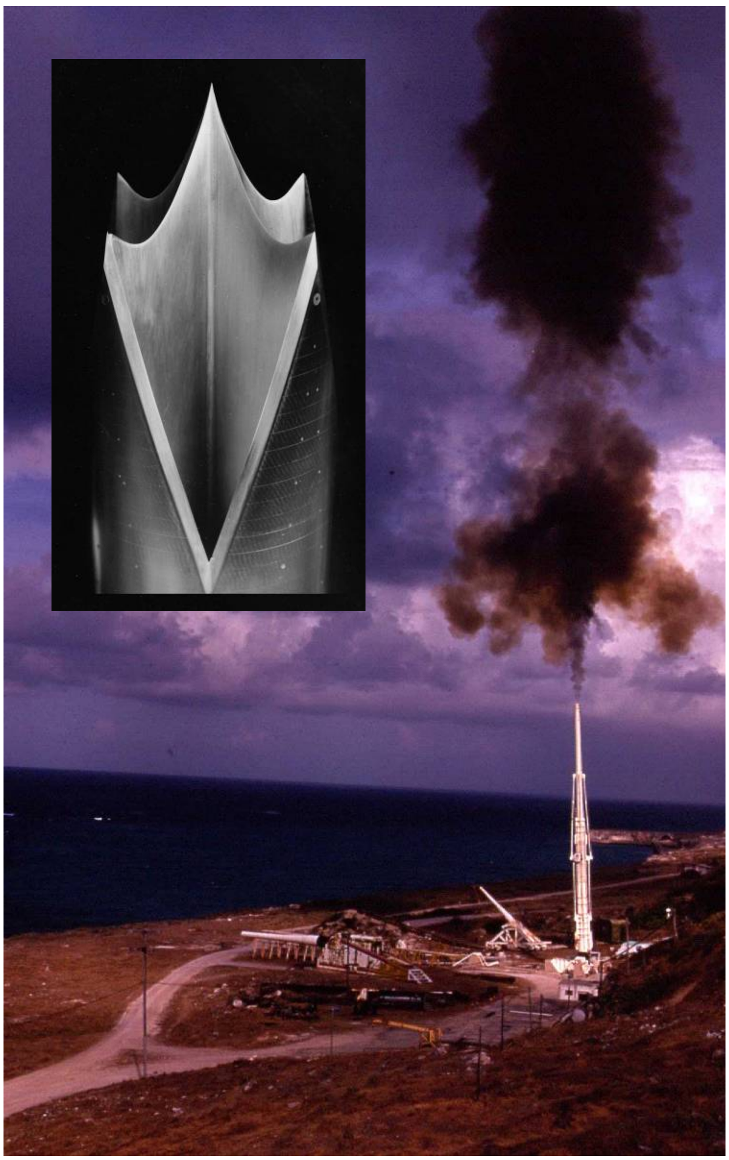
Textbooks in gas dynamics [e.g. Shapiro (1954); Emanuel (1994)] contain descriptions of simple flow-fields such as the flow behind a flat oblique shock, Prandtl-Meyer flow and flow over a circular cone at zero angle of attack. A design approach, based on *streamline tracing,* has been applied to generate *waverider* wing surface shapes from these, easily calculable, basic flow-fields. [Seddon and Spence, (1968)]. A similar technique, using planar or axial compressive flow-fields, leads to seemingly three-dimensional modular intake shapes called *wavecatchers*. The basic flows tend to be different for the two applications since the design goal of the wave-rider surface is to attain a high lift-to-drag ratio for the airplane, whereas the design goal for the wave-catcher surface strives towards a high-performance intake flow. Flat shock flow, Prandtl-Meyer flow and cone flow are used for 'external' flowfields on waveriders whereas flow in a conical duct and Busemann flow yield useful streamtube shapes for wavecatchers.

The JHU/APL SCRAM Missile [Keirsey *et al* (1965)] is based on tracing streamlines of flow in a contracting cone using calculations by the Method of Characteristics. The basic flow for this intake leads to design uncertainties stemming from the Mach disk at the axis of the basic flow.

JHU/APL SCRAM Missile

The four-module intake shown in the two views below is based on Busemann flow and wavecatching. Experimental results of this intake at Mach 8.3 are presented in Section 3.





A missile-type application of wave-catching is shown below, where four 90 deg modules of Busemann flow are combined back-to-back to capture a 360 deg, circular freestream tube. Such modules can be used singly or in combination to capture mass flow from free-stream tubes of arbitrary cross- sectional shape. Also, they are readily designed to have an open side that allows flow spillage during intake starting.

This intake was flown on a scramjet launched from a 16-inch gun in Barbados in 1972.

Four-module intakes were designed and studied also by Matthews and Jones (2003). Their designs were based on a conical contraction, similar to the JHU/APL approach, and also on a module wall with constant pressure. Flow calculations were done with the Method of Characteristics. The method of Characteristics cannot cope with the Mach disk and subsonic flow that always appear on the centre line for intakes with a finite strength leading edge shock so that some amount of judgment, respecting the centre-line flow structure, has to be applied in designing the intake's downstream surface.

It has been shown by Ogawa *et al* that separating boundary layers can exert first order effects on intake startability, making it important to include viscous flow influences in a final intake design. We exclude viscous flow effects only to allow enough space for treating the inviscid flow. The same applies to any considerations of real gas effects.

The first part of this paper concerns theory, performance and design of intakes derived from Busemann flow. The relevant Taylor-McColl equations are presented in Mach number variables resulting in some new information and insight of Busemann flow. Various conical flows are combined to yield novel intake/combustor flow paths. The second part deals with pseudo-steady flow **starting** in Busemann-flow-based module type intakes.

1.1 Intake Design – Dimensionality and Basic Flow

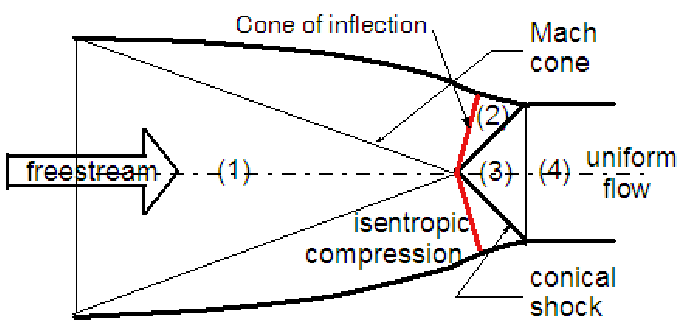
A great deal of understanding of intake flows comes about when one or more of the three physical space dimensions can be eliminated so that the flow at any location in the intake can be specified by fewer dimensions. Elimination is justified when flow properties do not vary with respect to the variable in question. This comes about when flow exhibits some degree of symmetry. For example, we obtain one-dimensional flow when it is reasonable to assume that there is no variation of flow properties in the two cross-stream directions. Flow properties then change only in the downstream direction. Algebraic expressions for normal shocks and isentropic flows become applicable and we can derive such notions as the Kantrowitz criterion for intake starting, offering a great amount of guidance and understanding at the intake design stage. Discarding even this single physical dimension leaves a lumped parametric representation allowing intake performance assessment in terms of a handful of numerical parameters. This, zero-dimensional representation is so useful that even when more detailed flow distribution information is available it is illuminating to use some averaging or integration of the detailed flow to obtain the equivalent zero-dimensional lumped quantities.

Eliminating the z-variable in the Cartesian (x, y, z) system results in *planar flow* in the (x, y)-system. In a spherical system (r, θ, ϕ) eliminating the circumferential angle, ϕ gives axisymmetric or *axial flow* in the (r, θ)-system.[3] If in axial flow it is reasonable to assume that there is no variation of flow properties in the radial (r) direction then the flow is strictly one-dimensional in the variable θ and the Taylor-McColl equations are obtained. Flows obeying the Taylor-McColl equations are called *conical flows.* Flow over a circular cone at

[3] These flows are sometimes called *two-dimensional* and *three-dimensional.* We will not use either of these designations since the axial flow does not need three, but only two, characterizing dimensions.

zero angle of attack is axial and conical. Flow inside a conical duct at zero angle of attack is axial only. Flow symmetry usually, but not always, appears because the bounding walls and the leading shock are symmetrical with respect to some symmetry variable(s).

Busemann [1944] demonstrated, analytically, the possibility of an axially and conically symmetric flow that starts as a supersonic and uniform free stream, compresses and contracts isentropically, finally passes through a conical shock wave to become parallel and uniform flow at a lower Mach number. The isentropic compression is contained between a Mach cone on the upstream side and a shock cone on the downstream side. Mölder and Szpiro [1966] proposed the Busemann flow as the basis for hypersonic air intake shape generation. A Busemann intake performance chart was presented which relates the intake's compression, contraction and efficiency. Using wave-rider methodology, Mölder and Romeskie [1968] presented the notion of selecting portions of the axisymmetric versions of Busemann flow to generate modular "wavecatcher" intake shapes with enhanced flow starting potential. Experimental results were presented for both full and modular (streamline traced) versions of the Busemann intake. Experimental performance of a full Busemann intake was compared by Mölder et.al. [1992] against a Prandtl-Meyer intake and an Oswatitsch type intake at a free stream Mach number of 8.33 and applications to flight vehicles were suggested by VanWie and Mölder [1992]. The above work has shown that Busemann flow which is axisymmetric, conical and bounded on the upstream by a Mach cone and on the downstream by a shock cone, does exist and that it has characteristics which make it suitable for use as a basis for the design of supersonic and hypersonic air intakes. Some new analytical features of Busemann flow are presented in Sections 2 and 3. Some new experimental results of Busemann flow starting are found in Section 4.



The traditional approach to intake design begins with the adoption of a gasdynamically simple, compressive flow consisting, typically, of a combined sequence of oblique shocks and isentropic flow fields. The resulting intake geometry is then examined in terms of its flow-starting potential. Some form of starting technique is applied and the intake is tested in a wind tunnel for performance as well as starting. A redesign is required if the flow fails to start. Modern, time-realistic, computer codes have made it possible to supplant much of the wind tunnel testing by CFD analysis. Nevertheless, the one-two-step-go-around approach is still applied in the design process.

A successful air intake design is based on most of the following considerations:

1) The intake should **start** easily, should not un-start and should operate with steady flow throughout the vehicle's flight envelope; starting should occur below the cruise Mach number;

2) The intake should meet specified capability criteria in that it should **contract, compress and reduce the Mach number** a desired amount;

3) The task specified in 2) should be performed **efficiently** as reflected in a minimal increase in entropy of the intake flow;
4) To avoid turning losses in the combustor and nozzle, the flow direction at the intake's exit should be **aligned** with the freestream; and the flow **profile** should be uniform;
5) It is desirable to deal with intake flows which are easily **analytically and computationally predictable**;
6) It is desirable not to have performance deterioration with **off-design** Mach number or angle of attack;
7) Mass flow loss due to **overboard spillage** should be minimal and, if necessary, confined to the time period of intake starting;
8) The **external drag** should be minimal;
9) The **side forces** due to angle of attack should be as small as possible since these forces would, in general, be aerodynamically destabilizing because of the intake's forward location;
10) The intake should withstand **high acceleration, high internal pressure and high heat transfer** by avoiding the use of **variable geometry**.

Well-advanced intake development programs, focusing on a specific design, examine flow quality at the intake's exit plane and off-design performance as it affects the combustor performance. It is possible that, depending on the mode and manner of fuel injection and combustion, a uniform flow profile is not desirable.

CFD studies [Molder et al (1992)] have shown that a given Busemann contour seems to produce a uniform exit flow at two distinctly different freestream Mach numbers. This discovery makes the intake suitable for use on dual-cycle engines that operate with subsonic or supersonic combustion, depending on the freestream Mach number. This possibility needs further analysis and development.

It is conceivable that a low efficiency intake, that starts at a low Mach number, provides enough engine thrust for the missile to take over early from its rocket or turbojet booster, thereby providing a better missile range than a high efficiency intake that starts at a higher take-over Mach number. This illustrates the fact that intake efficiency, as well as the other intake performance parameters, are not the final arbiters in intake selection for a specific missile mission. Careful examination of such compromises and trade-offs applies to other engine components as well so that on-design conditions may not occur at optimal values of any single performance parameter but rather at a set of design compromises that optimize the overall mission goals.

In the design approach proposed here, a startable intake is chosen from a subclass of startable intakes that is part of a class of high-performance intakes. Steady-state startability is determined by the EKD-criterion [Eggink (1943), Kantrowitz and Donaldson (1945)]. High-performance intakes are of the Busemann type [Busemann (1944), Mölder and Szpiro (1966)].

Aerodynamics mandates that the critical design issues are found at the downstream end of the intake flowpath. This is where the starting process terminates, this is where shocks and boundary layers interact and this is where combustor conditions are specified. It makes sense to start the design from considerations at the downstream end so that the design process is not so much a circle as it is an iteration where the downstream, desired conditions, are used to iterate to a desired freestream Mach number.

1.2 Intake Starting Issues

A converging duct in supersonic flow, such as an air intake, can support two distinctly different flow configurations for the same free-stream Mach number. One configuration has a bow shock in front of the intake that diverts some flow overboard and, in this case, the internal flow is subsonic. This flow is termed *sub-critical* and the intake is *unstarted*. The second possible configuration has no bow shock, no overboard spillage and is supersonic throughout [Shapiro (1954)]. This *supercritical*, or *started*, flow is required for proper operation of the engine. High losses of total pressure and mass flow, associated with unstarted flow, make flow starting mandatory for efficient engine operation. Starting requires that the normal shock, in front of the intake, moves downstream into the intake (is *swallowed*) and that a stable hypersonic/supersonic flow establishes throughout the converging portion of the intake. There are several ways to start an intake, including such traditional quasi-steady techniques as **overspeeding**, use of **variable geometry**, **overboard spillage**, mass spillage via intake **wall perforations**, as well as some new techniques specifically harnessing **unsteady flow** effects. Intake starting at high Mach numbers has been challenging using the traditional techniques; over-speeding is not applicable for high Mach number flows; variable geometry introduces sealing problems for large changes in intake wall geometry. Mass spillage through wall perforations was first introduced by Evvard and Blakey in 1947. They suggested drilling perforations in the intake wall to spill the mass that the throat could not ingest without choking. Unacceptable mass flow losses are present if the holes are not closed after the intake has started, introducing mechanical complexity. For perforated intakes, there is a need to develop detailed information on flow extraction from subsonic and, to a lesser extent, from supersonic streams, through slits, slots and holes of various shapes.

Experiments, and computations, [Molder and Romeskie (1968)] have shown that, if the freestream flow is **pulsed**, such as at the start of a wind tunnel nozzle, intakes with contraction ratios as high as 20 will start. This type of intake starting usually works in combination with, and is aided by, an initial low pressure in the intake duct - a fortuitous and happy circumstance for testing in pulsed and vacuum-driven wind tunnel facilities where, because of the short running times, it would be very difficult to start the intake by other means, such as changes in intake geometry. For flight vehicles, however, such pulsed flow conditions or low initial pressures are not readily available and one has to attain intake flow starting, under steady freestream conditions, by some other means.

The absolute necessity of flow starting places serious constraints on intake design.

Designing a high-compression intake for **starting** presents, not only a unique, but also one of the most difficult challenges in the application of compressible flow gasdynamics. The starting event is brief - lasting of the order of one second. Conventional, long-term, aerodynamic efficiency is unimportant whilst effectiveness is paramount. Even though some starting techniques appear analyzable with the quasi-steady flow assumption, the starting flow is fundamentally unsteady and, if simulated computationally, must be simulated with a time-realistic computer code. Prediction of multiple, time-asymptotic, equilibrium flow states requires the use of a code that depicts shock wave geometry and motion with high-fidelity. Although the inviscid intake design may be based on one or more of the simple, exact flows, the starting flow is most likely three-dimensional, possessing no simplifying degrees of symmetry. Relatively strong starting shocks, and their interaction with the boundary layer, call for an accounting of shock-boundary-layer effects. The 3D time-dependent, viscous flow must be recognized, understood and harnessed. Design for starting will be strongly affected by the rest of the engine, the vehicle and the mission and vice versa. This is an intriguing area of gas dynamics, loaded with challenges, where a successful intake starting technique will employ a synergistic combination of 3D analysis methods, both steady and unsteady. Design for flow starting calls for both aerodynamic and mechanical ingenuity. It is a struggle between having enough contraction for high engine performance yet not too much contraction to prevent intake starting. Intake efficiency is unimportant during starting because the starting process is short.

There is a need to develop effective methods of intake flow starting that are not deleterious to the steady-state performance of the intake, with the stark realization that a superbly capable and efficient on-design intake flow is worthless if the intake fails to start. Starting is a go-no-go proposition; it must work, there is no `half-way', not even an `almost'. **We present a design approach that begins by specifying the exit conditions of a high-efficiency, startable intake.**

2.0 DESIGN FOR HIGH PERFORMANCE

This section describes the characteristics of Busemann flow that make it suitable as a basis for air intakes. The theory behind Busemann flow is outlined. Results are given for Busemann flow as applicable to intake performance in terms of intake contraction and efficiency. Possible geometric variants and wavecatching are described.

2.1 The Choice of Basic Intake Flow

Oswatitsch, by classical variance techniques, demonstrated that an intake flow, containing a sequence of plane, oblique shocks, attains its best performance when the shocks are of equal strength. Shocks of infinitesimal strength (Mach waves) produce isentropic flows with no thermodynamic losses and 'ideal' intakes. These very simple flows, with sequential plane, oblique shocks, led to many practical intakes, the Concorde SST being an example. Intakes based on cone flow have been common on fighter airplanes, the early MIGs and the SR-71 being examples. Conceptually, reversing the flow in a converging-diverging wind tunnel nozzle leads to an intake shape with compressive isentropic flow. However, such a shape produces a sonic or subsonic exit flow that is not suitable for scramjets. The shape of a perfectly isentropic hypersonic to supersonic "nozzle-intake" has not been formulated. If a small shock loss is tolerable then conical flow offers a suitable shape based on near-isentropic Busemann flow.

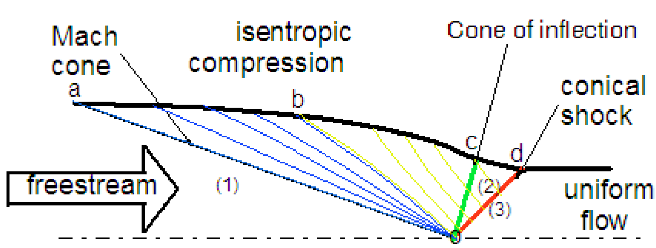
2.2 Busemann Flow

A preferred geometry for a scramjet combustor is a circular cross-section duct because of its superior ability to withstand both heat and pressure loads. Frictional losses are also at a minimum for such a duct since a cylinder has the smallest surface area for a given cross-sectional area. This leads to a cylindrical (axially symmetric) geometry being desirable also for the intake that is attached to the front of the combustor duct. The same circular intake exit geometry is demanded by a gas turbine engine, this time because the axial compressor face is circular. In design selection of a suitable aerodynamic flowpath geometry, the requirement of high aerodynamic efficiency leads to intake flow types where any isentropic compression precedes shock compression so that the shock can occur at the lowest possible Mach number. Towards these ends, it is wise to study an axisymmetric flow and it is entirely fortuitous that axisymmetric, conical, Taylor-McColl flow provides a streamtube shape that satisfies the above intake design requirements, both geometric/structural as well as aerodynamic. In recognition of A. Busemann's [Busemann, 1944] work on such streamtube shapes, they are called Busemann flows and Busemann intakes.

Enforcing conical symmetry for Busemann flow leads to flow quantities being constant on cones whose apecis all lie on the same point and whose axes are all parallel to the free stream. Imposing conicality restricts considerations to this specific class of flow while, at the same time, offering great simplicity in flow analysis where a wide variety of intake surfaces is available for selection - surface shapes that yield both a high compression and a high efficiency. Disappearance of the radial dimension as an independent variable, in conically symmetric flow, permits the depiction of results on the single remaining spatial variable – the conical angle. Furthermore, the use of conical flow means that all shocks are also conical and therefore of constant strength at any given angular position. The flows are then not only uniform but also irrotational – generally, a desirable feature for flow that leaves the intake to enter a combustion chamber. These features of conical flow and, in particular, Busemann flow, which is by nature an internal flow, make the Busemann streamline shape an attractive candidate for an air intake of a hypersonic flight vehicle's engine.

In Busemann flow, compression from the high freestream Mach number is initially isentropic. Only at the lowest Mach number does the flow pass through a shock. The shock is weak and produces a downstream flow, which is irrotational, uniform and parallel to the free stream. High stream-wise pressure gradients occur *in* the flow as opposed to at the walls. High overall compression and substantial Mach number reduction is attained at high efficiency. As an example: A Busemann intake reduces the Mach number from 8.33 to 2.8 with a total pressure recovery of 91%. In choosing a particular design, one can start by specifying the desired exit conditions and the efficiency – an approach suitable for preliminary design selection. Alternatively, one can start by selecting a shock pressure ratio low enough to keep the boundary layer attached at the shock impingement point and then proceeding by considering all intakes satisfying these two conditions. Another virtue of the Busemann design approach is that the surface contours and intake operating conditions are very easily calculable, allowing ready perusal of multiple design options.

A schematic of Busemann flow contours are shown below. Uniform, parallel freestream flow, state (1), from the left, is isentropically compressed from a free stream Mach cone up to the shock cone, state (2), and then the flow passes through the conical shock to become uniform and parallel flow at state (3). The flow is both axially and conically symmetric and irrotational throughout. In passing from state (1) to state (3), the flow is contracted and compressed and there is a loss of total pressure at the shock. Detailed examination of the shape of the Busemann streamline has shown that the upstream part of the streamline is curved towards the centre line and that this is followed by a downstream part that is curved away from the axis. These two portions are then separated by an inflection point. The heavy green line indicates a cone that contains inflection points of all the Busemann streamlines. This inflection point cone has special significance to the starting of supersonic flow in the intake.



There are several features inherent in Busemann flow which make it a suitable candidate when selecting a basic flow for an air intake:

1) Starting the integration of the T-M equations from the first quadrant - at either a strong or weak conical shock, with uniform and parallel downstream flow, always produces a uniform and parallel freestream, at a higher Mach number, in the second quadrant - a necessary condition for the flow to be applicable to air intakes;

2) A conical normal shock with its base at the inflection point (c) of the Busemann surface and its apex at the focus (o) marks the beginning of internal contraction; the shock's area and the exit area determine the startability of a Busemann intake with mass spillage;

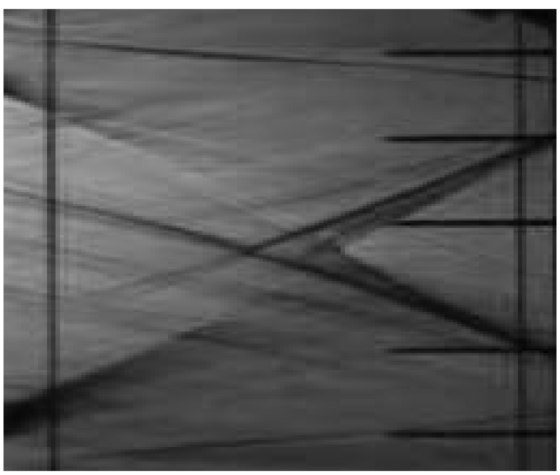
3) A high degree of compression, at high Mach number, takes place isentropically over the surface abc; shock compression occurs at the lowest possible Mach number; this leads to a high-performance intake;

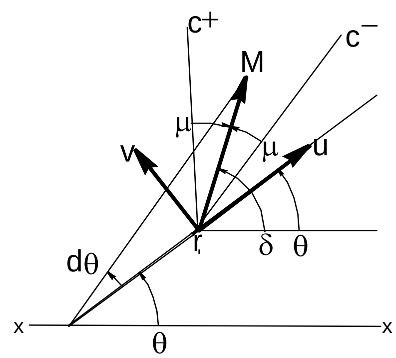
4) The Busemann streamline is easily adaptable to wavecatching, leading to modular intakes with arbitrary capture stream tube shapes;

5) Blue compression waves, emanating from the surface segment ab, coalesce to the origin; a novel feature in axial supersonic flow;

6) Orange characteristics, emanating from the surface segment bcd impinge on the front of the shock;

7) The free-standing, conical, 'Busemann shock' seems unreal since it is not supported by a physical cone; however it is compatible with T-M flow and the oblique shock relations; and has been experimentally verified as shown in the schlieren picture above, [Mölder et al, 2011].





2.3 The Taylor-McColl Equations

The Taylor-McColl equations govern inviscid compressible flow that is both axially and conically symmetric. Their most well-known application has been to the flow over a circular cone at zero angle of attack [Sims, (1964)]. Application to three other flows, including Busemann, are found in Grodzovskii (1959) and Mölder (1967).

Flow that is both axially and conically symmetric is best described in spherical polar coordinates (r,θ) where r is distance measured radially out from the origin and θ is the angle measured from the downstream direction. In all cases the origin is at the apex of the conical shock, on the centre line of symmetry (xx). The flow velocity components in the radial and angular directions are designated as U and V. Drawing similar triangles along the streamline in the figure on the right gives the streamline equation:

$$dr / d\theta = rU / V \tag{2.0}$$

The original Taylor-McColl Equation is a non-linear, second order total differential equation with the spherical polar angle, θ, as independent variable and the radial flow velocity, *U*, as dependent variable [Anderson 1982, Emanuel, 1994].

$$\frac{\gamma-1}{2}\left[1-U^2-\left(\frac{dU}{d\theta}\right)^2\right]\left[2U+\frac{dU}{d\theta}\cot\theta+\frac{d^2U}{d\theta^2}\right]-\frac{dU}{d\theta}\left[U\frac{dU}{d\theta}+\frac{dU}{d\theta}\left(\frac{d^2U}{d\theta^2}\right)\right]=0 \qquad (2.1)$$

This is the model equation that governs steady, axisymmetric, conical flow of a perfect gas. No explicit algebraic solution has been found; nor are there any numerical schemes for solution of the **second order** equation (2.1) as given above. However, the equation can be converted to two **first order** equations, (2.2) and (2.3), at the price of acquiring the additional dependent variable, *V*. The two equations are now amenable to standard numerical solution methods. Most of these solutions have been done with boundary conditions applicable to cone flow [Sims, (1964), Anderson, (1982) Emanuel, (1994)].

2.3.1 The First Order Taylor-McColl Equations

The first-order versions of equation (2.1) are the momentum equations, in polar coordinates, in the *r* and θ directions,

$$dV/d\theta = -U + \frac{a^2\left(U + V\cot\theta\right)}{V^2 - a^2} \qquad (2.2)$$

$$dU/d\theta = V \qquad (2.3)$$

where *a* is the speed of sound that can be written in terms of the velocities and the total conditions through the energy equation, [Thompson, p.488, 1972]. The second of these equations is also the irrotationality condition, implying that conical flows are necessarily irrotational. Explicit reference to the speed of sound and total conditions can be circumvented if the equations are recast so as to have the radial and angular Mach number components as dependent variables in place of the corresponding velocity components. The boundary conditions, when expressed as Mach number components at the up- and downstream sides of conical shocks are then applicable directly to the solution of the equations. Also, total conditions, which have no influence on the Mach number solution, do not have to be invoked.

2.3.2 Mach Number Variables

The Taylor-McColl (T-M) Eqns.(2.2 and 2.3) have been recast in terms of the radial and angular Mach numbers *u* and *v*, where *u* = *U/a* and *v* = *V/a* and *a* is the local sound speed:

$$\frac{du}{d\theta} = v + \frac{\gamma-1}{2}uv\frac{u+v\cot\theta}{v^2-1} \qquad (2.4)$$

$$\frac{dv}{d\theta} = -u + \left(1+\frac{\gamma-1}{2}v^2\right)\frac{u+v\cot\theta}{v^2-1} \qquad (2.5)$$

These two equations seem more complicated than their parents, (2.2) and (2.3). However, it will be shown that the use of Mach number components *u* and *v* leads to meaningful and useful physical interpretations from Eqns. (2.4) and (2.5). Note that the sound speed no longer appears explicitly in the equations; neither do any total conditions.

The streamline equation is:

$$dr / d\theta = ru / v \tag{2.6}$$

The flow Mach number is:

$$M = \sqrt{u^2 + v^2}$$

Having the T-M equations in this form reveals their singular nature at $v = \pm 1$ where the singularity is caused by the $(v^2 - 1)$*-term* in the denominators above becoming zero.[4] The singularity appears when the angular Mach number component becomes sonic. This occurs when a radial and a Mach wave coincide. Absence of any explicit reference to total conditions, as well as the sound speed, leads to a more straightforward application of the boundary conditions. A standard, fourth-order Runge-Kutta scheme has been used to integrate the Mach number equations (2.4) and (2.5). The solutions are identical, to eight decimal places, to similar solutions of (2.2) and (2.3) in the velocity variables. Previous reference to the T-M equations in Mach number form has not been found in the literature.

2.3.3 Busemann Flow Boundary Conditions

Busemann flow and its streamline shape are calculated from the T-M equations (2.4) and (2.5). These equations are integrated with respect to θ from the front of the conical shock (station 2) to the free stream (station 1). To do so requires the starting values (boundary conditions): u_2, v_2 and θ_2. These have to be specified in such a way that the flow downstream of the shock will be parallel to the free stream; this is the most common requirement of flow entering the combustor. This condition must be applied to find the appropriate combination of u_2, v_2, and θ_2. Using the Mach number in front of the shock, M_2, and the aerodynamic shock angle, θ_{23}, the radial and angular Mach numbers in front of the shock are:

$$u_2 = M_2 \cos\theta_{23} \tag{2.9}$$

$$v_2 = -M_2 \sin\theta_{23} \tag{2.10}$$

The flow deflection through the shock is obtained from the equation relating Mach number, shock angle and flow deflection through the shock [Anon. NACA Rep. 1135, 1953, Eqn. 139a]:

$$\tan\delta_{23} = \frac{2\cot\theta_{23}\left(M_2^2 \sin^2\theta_{23} - 1\right)}{2M_2^2\left(\gamma + 1 - 2\sin^2\theta_{23}\right)} \tag{2.11}$$

[4] Such singularities are discussed by Dadliz [1946], Mölder [1967] and Rylov [1990]. Their appearance, in any given flow should be taken as a warning that whatever symmetry assumption(s) have been made may not hold in the physical airflow. Conical boundary conditions do lead to non-conical flow in some cases.

The angular location of the shock which is the starting value for the variable of integration, is then:

$$\theta_2 = \theta_{23} - \delta_{23} \tag{2.12}$$

Equations (2.4) and (2.5) are then numerically integrated from θ_2 *to* $\theta_1 = \pi - \mu_1$. Since θ_1 is not known *a priori*, the integration is continued until the vertical or cross-stream Mach number $(u \sin\theta + v \cos\theta)$ becomes zero, indicating that the free stream has been reached. Note that, prior to integration, we could calculate the intake's efficiency, using the total pressure ratio as measure (figure below),

$$p_{t3} / p_{t2} = \left[\frac{(\gamma+1)k^2}{(\gamma-1)k^2+2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma k^2 - \gamma + 1} \right]^{\frac{1}{\gamma-1}} \tag{2.13}$$

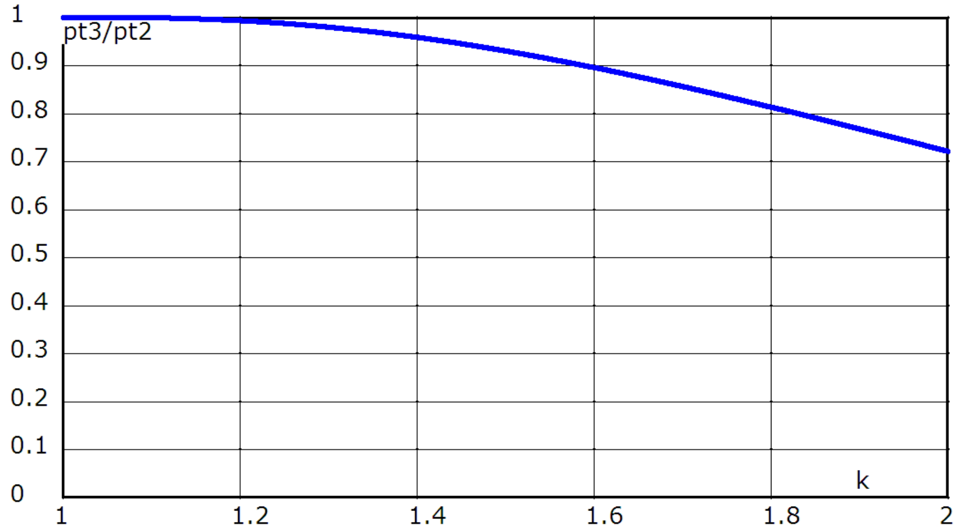
and the exit Mach number (figure below),

$$M_3^2 = \frac{(\gamma+1)^2 M_2^2 k^2 - 4(k^2-1)(\gamma k^2+1)}{[2\gamma k^2 - (\gamma-1)][(\gamma-1)k^2+2]} \tag{2.14}$$

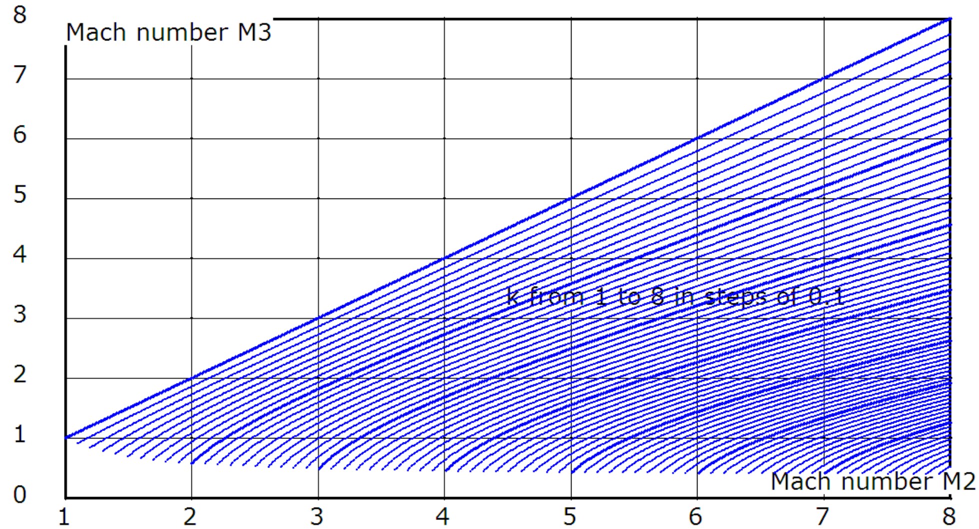
where $k^2 = M_2^2 \sin^2\theta_{23}$. In fact, we could **prescribe** a desired efficiency; calculate k from Eqn. (2.13); **prescribe** the downstream Mach number M_3, calculate M_2 by inverting (2.14); then $\theta_{23} = \sin^{-1}(k/M_2)$, $u_2 = M_2\cos\theta_{23}$ *and* $v_2 = M_2\sin\theta_{23}$. After this, θ_2 and δ_{23} are found as above and the integration performed until $(u + v \cot\theta) \geq 0$. The ability to specify the downstream Mach number and an intake efficiency, **before doing the integration**, makes this approach particularly suitable for preliminary intake design selection. Instead of the total pressure ratio (Eqn. 2.13) we could prescribe a static pressure ratio across the shock as, [NACA 1135, Eqn. 128],

$$\frac{p_3}{p_2} = \frac{2\gamma k^2 - \gamma + 1}{\gamma + 1}$$

A value of p_3/p_2 can be prescribed such that, with M_2, the shock will not cause boundary layer separation at the point where it meets the wall. Note, however, that *all is not roses,* since the integration, beginning with k and M_2, yields a free stream Mach number that may not be the desired one. An iteration on the input conditions, pt_3/pt_2 and M_3 or p_3/p_2 and M_3 has to be performed to arrive at the desired design Mach number of the flight vehicle. This inconvenience is the direct result of, and the price paid for, the convenience and simplicity achieved by assuming a conical flow. At the free stream condition an infinite number of different intakes are possible at any specified Mach number. This is in agreement with the appearance of the singularity, in the T-M equations, at the freestream condition, which makes it impossible to start the integration at a specific freestream Mach number – an infinite number of streamlines are possible, proper boundary conditions cannot be specified. However, an *a priori* selection of Mach numbers is possible from pre-calculated Busemann flows - as shown in the next section.



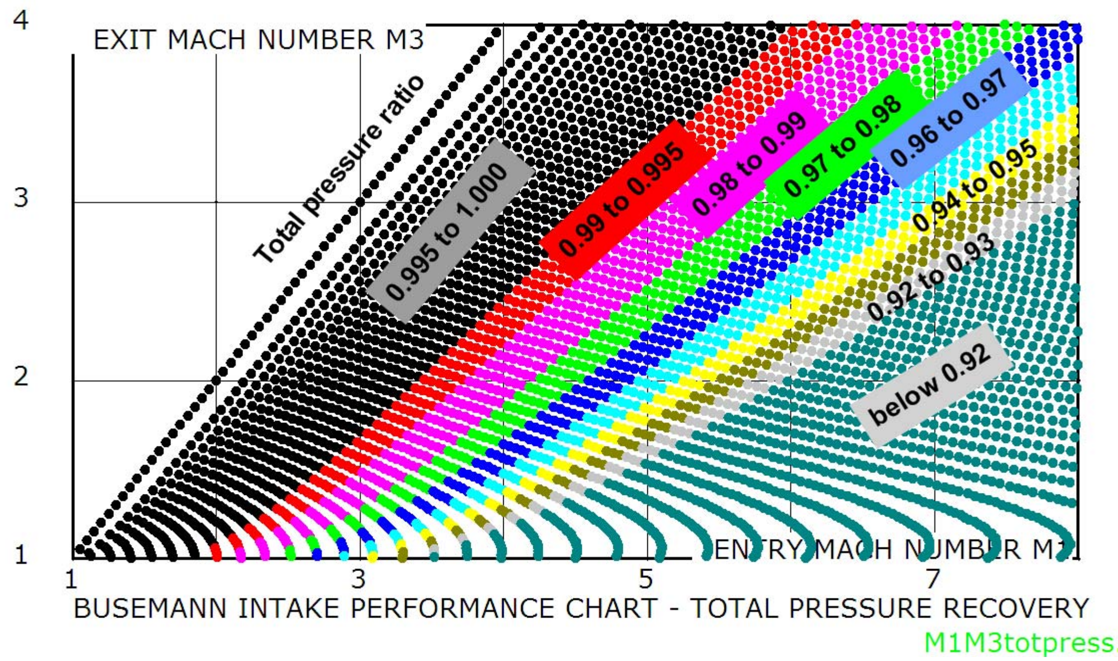
Total pressure recovery behind oblique shock as function k=M2sin(theta)



Mach number behind oblique shock M3 as function of Mach number in front M2 with k=M2sin(theta) as parameter

2.4 Anatomy of Busemann Flow; The Performance Map

Proceeding with the integration of the TM-equations from the initial conditions, as chosen above, produces a free-stream Mach number M_1. The results of many such calculations are presented in this section where, in each case, a value of M_2 is selected, in our case between 1 and 8 and k is cycled from 1 to M_2. For each M_2 and k the total pressure ratio and M_3 are calculated; integration of the T-M equations then leads to the freestream at M_1 and a point is plotted on a graph of M_1 vs. M_3 with p_{t3}/p_{t1} as parameter, determining the point's colour.



Every point in this figure represents a Busemann intake calculation from the downstream shock to the freestream. This graph can be used to select a Busemann intake design based on the entry and exit Mach numbers and the total pressure ratio. Any two of these parameters can be used to determine the third. For example, an intake that reduces the freestream Mach number from 7 to 3 does so with a total pressure recovery of 0.95.

2.5 Inflection Point on the Busemann Streamline

An equation for the curvature of the T-M streamline is derived to show that the Taylor-McColl streamline can have points of zero and infinite curvature. The Busemann streamline has two points of zero curvature where one of these points has significance in the starting of a Busemann-type intake. The conical surface containing all inflection points in a typical Busemann flow is shown in green in the sketch above. To derive the properties of the flow at the inflection cone we note that the defining equation of the T-M streamline is,

$$dr / d\theta = ru / v \tag{2.15}$$

where u and v are the radial and angular components of Mach number as used in the T-M equations (2.4) and (2.5). Taking another θ-derivative of (2.15) gives,

$$\frac{d^2r}{d\theta^2} = -r\frac{u}{v^2}\frac{dv}{d\theta} + \frac{r}{v}\frac{du}{d\theta} + \frac{ru^2}{v^2} \tag{2.19}$$

In polar coordinates the curvature of a planar curve is [Kreyszig, p.34, 1991],

$$D = \left(\frac{\partial \delta}{\partial s}\right) = \frac{r^2 + 2\left(\dfrac{dr}{d\theta}\right)^2 - r\dfrac{d^2 r}{d\theta^2}}{\left(r^2 + \left(\dfrac{dr}{d\theta}\right)^2\right)^{3/2}} \tag{2.20}$$

Eliminating the derivatives of r with (2.15) and (2.19) gives,

$$D = \frac{r^2 + 2\left(ru/v\right)^2 + r^2 \dfrac{u}{v^2}\dfrac{dv}{d\theta} - \dfrac{r^2}{v}\dfrac{du}{d\theta} - \left(ru/v\right)^2}{\left(r^2 + r^2u^2/v^2\right)^{3/2}} \tag{2.21}$$

In this expression the derivatives $dv/d\theta$ and $du/d\theta$ are given by the T-M equations (2.4) and (2.5) so that the curvature can be written,

$$\boxed{D = \frac{uv\left(u + v\cot\theta\right)}{r\left(v^2 - 1\right)\left(v^2 + u^2\right)^{3/2}}} \tag{2.22}$$

This equation gives the curvature of the T-M streamline in terms of the polar coordinates, r and θ, and the radial and polar Mach number components, u and v. A number of very interesting and important features, about the T-M streamline, become apparent from an examination of its curvature as given by (2.22):

1) D is inversely proportional to r so that when $r \to 0$ then $D \to \infty$. This means that streamlines near the origin of T-M flows are highly curved. This is a necessary condition for flow over a cone, where flow near the tip and just aft of the conical shock, has to rapidly adjust to the inclination demanded by the cone since the flow deflection produced by the conical shock is insufficient for the flow to be tangent to the cone surface. Similar, highly curved streamlines are to be expected near the origin of Busemann flow. Conical flow is not conically symmetric (i.e. independent of r) when it comes to gradients of its dependent variables, such as streamline curvature because the dependence is inversely proportional to r. This extends to other flow property gradients, such as pressure, as well.

2) There is an asymptotic condition, (D = 0) in the T-M streamlines at $v = 0$. For flow over a cone, $v = 0$ at the cone surface. This confirms that the streamlines become asymptotic to the cone surface as they approach the surface. There is no $v = 0$ or $u = 0$ asymptotic condition in Busemann flow.

3) When $u = 0$ then $D = 0$. This means that the streamline has a point of inflection at the place where the radial Mach number is zero. For flow over a cone the condition $u = 0$ never occurs, so the streamlines are curved monotonically positive. However, for Busemann flow there is a location, θ_o, where the streamline changes from being concave towards the axis (negative curvature) to being convex (positive curvature). Numerical integrations of the T-M equations have shown that θ_o always lies in the interval θ_2 to $\pi/2$ (first quadrant) somewhat upstream of the Busemann shock as shown by the green line in the sketch of Busemann flow above. Every Busemann streamline has an inflection point and these points form a conical surface. **At this angular location of the inflection the flow is everywhere normal to the inflected flow cone surface and a conical normal shock can be placed**

here since the Mach number is supersonic! The shock could be coaxed into taking up this position by allowing enough mass spillage to occur upstream of the inflection location, [Fabri, 1958] and by restricting the downstream contraction to that allowable by the Kantrowitz criterion for flow starting. Flow just downstream of the conical normal shock is inclined towards the axis. This is tolerable everywhere but not right at the axis since at the axis the flow must be aligned with the axis. This ($r \to 0$)-type singularity is similar to the cone-tip singularity described above; its existence in the idealized form has not seen confirmation by experiment or CFD. If the contraction downstream of the conical normal shock surface does not lead to choking, then the shock would move downstream and the intake **would start spontaneously**. This feature has not been appreciated for Busemann flow. It has significance in the design of self-starting supersonic/hypersonic air intakes. It is a conical and axisymmetric example of the starting criterion proposed by Kantrowitz for one-dimensional flow, embodying the same principle of flow choking downstream of a normal shock where, in our case, the normal shock has a conical shape. The normal shock at the inflection point and a choked exit conform to the Kantrowitz criterion for flow starting in the internal contraction downstream of the inflection. This flow situation is used in Section 3 to determine intake startability of intakes based on strong shock flow.

4) There is a point of inflection also when $(u + v\cot\theta) = 0$. The quantity $(u + v\cot\theta)$ is the component of Mach number normal to the flow axis. For Busemann flow it is zero only where the Busemann flow joins the free stream. Thus the leading edge of the Busemann flow has not only zero deflection but also zero curvature. Aerodynamically this means that the leading edge wave is neither compressive nor expansive but is a simple Mach wave. The fact that the entering free stream flow is neither deflected nor curved by the Busemann leading edge means that the leading edge of a hypersonic air intake, based on Busemann flow, is totally ineffective in producing compression. This provides a clear incentive to **truncate some length of the leading edge** surface so as to decrease viscous losses without incurring serious inviscid flow losses.

5) When $v \to \pm 1$ then $D \to \infty$; the curvature is infinite and the streamline has a cusp or a corner. This indicates a *singularity* or a *limit line*. Neither cone nor Busemann flow exhibit such a limit line.

6) The quantity $(v^2 + u^2)^{3/2}$, appearing in the denominator of (2.22), is just M^3. It is always a positive quantity for all flows and has no drastic characterizing effect on D except to force streamlines to lose their curvature (to straighten out) at hypersonic speeds.

The **inflection point** has been discussed above because it is an interesting feature of Busemann flow and, more importantly in the intake starting context, because the ray from the origin to the surface is normal to the surface at the inflection point. There is no obvious reason why these points coincide but it implies that a conical, normal shock can exist at the inflection point. This is further reinforced by an obscure result of Curved Shock Theory that says that a normal shock cannot sit on a surface with non-zero curvature, implying that the zero curvature at the inflection point provides a ‘comfortable’ location for the normal shock. For intake starting, a normal shock has to be established at the inflection point and it must there have a local startability index greater than one for the shock to move downstream to start the intake.

2.6 Geometric Variations of Busemann Intake Flows

In this section we present the possibility of joining Busemann flow to cone flow and joining several Busemann flows in tandem to obtain variations in intake performance and flow path geometry. Concatenation of Busemann flow to other types of flow is facilitated by the fact that Busemann flow is bounded on the up and downstream sides by uniform flow - it joins one uniform flow to another. We introduce the notion of wavecatching which at once allows: a) freedom in choosing freestream capture cross-section shapes; b) possibility of sweeping the leading edges and c) overboard mass spillage and great enhancement of startability.

2.6.1 Variations of Basic Flow

The basic axisymmetric Busemann flow accommodates several variations that, because of their shape, have pertinence to intake design:

a) The Busemann streamline leading edge has zero deflection as well as zero curvature. A good portion of the leading edge and its downstream surface do very little in compressing the incoming flow. Eliminating this leading edge surface area decreases boundary layer development and losses. However, the new leading edge now has a finite deflection bringing about leading edge shock losses. There is clearly an optimum amount of leading edge truncation and a design challenge involving **leading edge blunting** and its effects on intake performance and edge integrity.

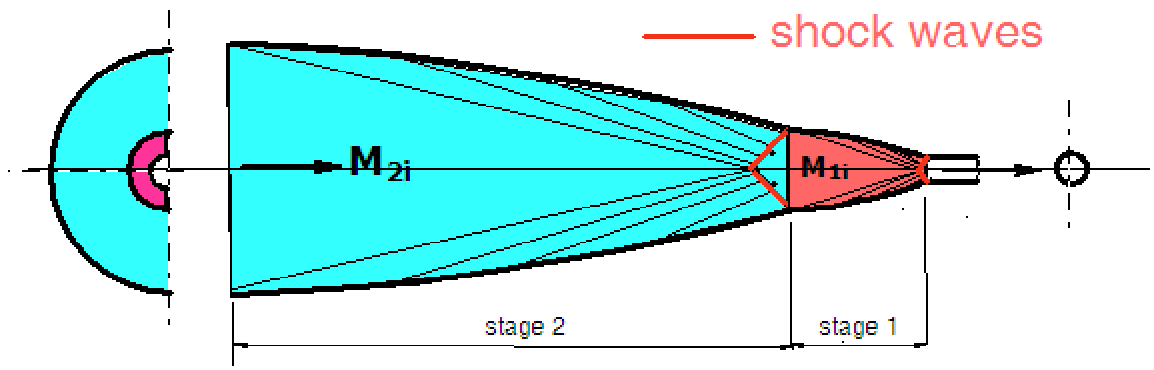
b) Flow at the exit of the basic Busemann flow can be further compressed in a subsequent Busemann intake. This notion of **staged intakes** leads to the possibility of starting each stage in sequence where each stage contracts no more than is required to permit spontaneous starting, resulting in the start of the assembled stages where the assembly would not start as a unit.

c) Since the exit flow from a Busemann intake is uniform and parallel it may be thought of as a free stream and a **cone** may be placed on the centre line at the exit. The exit flow is then further compressed by the cone into an **annular passage**.

d) Conical symmetry of the Busemann flow allows any calculated Busemann streamline to be scaled with respect to distance from the origin. Such scaled streamlines can be placed adjacent to one another to form a streamtube of arbitrary cross-section. Streamtubes, replaced by solid surfaces, form **modular intakes.**

2.6.2 Staged Busemann intakes

The exit flow from a single Busemann intake is uniform and parallel to the flow at the entry. This exit flow may be compressed further by placing another Busemann intake just downstream so that it ingests the exit flow from the upstream intake. Two or more intakes connected in a stream-wise sequence is termed *staging*. The figure shows a Two-Stage Busemann Intake.



Two-Stage Busemann Intake

Stages are numbered from downstream-to-upstream. Thus stage 1 has an entry Mach number of M_{1i} and stage 2 has and entry Mach number M_{2i} which is also the free stream Mach number for this two-stage intake. The Mach number at the exit of the first stage is M_{1e}. The calculation begins by assigning values to any two of the three variables n, M_2 and k.

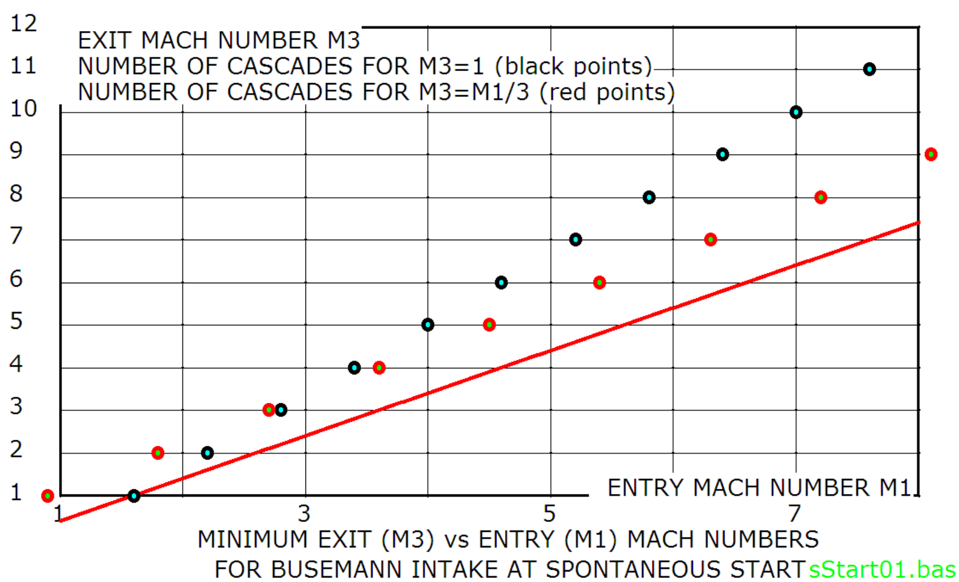
Number of stages n	1	2	4
M_2	3.72	3.33	3.12
k	1.58	1.22	1.084

The above input values have been iterated to produce the same Mach number reduction from 8 to 3 in each of the three staged intakes.

Number of stages	n	1	2	4
Entry Mach number	M_1	8.00	8.01	7.99
Exit Mach number	M_3	3.00	3.01	2.99
Contraction ratio	A_1/A_3	41.8	43.8	44.9
Compression Ratio	p_3/p_1	243	258	266
Shock pressure ratio	p_3/p_2	2.56	1.35	1.02
Total pressure loss	$100(1-p_{t3}/p_{t1})\%$	8%	2%	0.3%

The above table shows the performance of one, two and four-stage Busemann intakes. So as to provide a fair comparison, the *capability* has been made the same for the three intakes in that the Mach number is reduced from 8 to 3 for each intake. Contraction and compression ratios are very similar. There is a significant difference in the individual shock pressure ratios. Within each staged intake the shock strengths are equal. For the one-stage intake, the shock pressure ratio of 2.56 is strong enough to cause boundary layer detachment whether the boundary layer is laminar or turbulent. For the two-stage intake a turbulent boundary layer will remain attached but a laminar layer will separate. For the four-stage intake the boundary layer will not separate at the shock waves. This illustrates one of the advantages of staging. The other advantage shows in the *efficiency* as measured by the per cent total pressure loss. The one-stage intake has a loss of 8% whereas the four-stage intake is almost isentropic at 0.3%. The overall efficiency loss has been decreased markedly by spreading it over 2 and 4 stages, in each case.

Staging can be used also for making an intake where **each stage individually satisfies the Kantrowitz criterion for starting**. The individual stages would then start spontaneously in a downstream sequence to give the overall performance of an intake that would not start as a single unit. The black symbols in the figure below show how many concatenated stages are required to bring the flow to choking from any given freestream Mach number. Each stage has been calculated as a Busemann intake feeding its exit flow into another Busemann intake just downstream. For example, 10 stages would be required to choke the flow from Mach 7. It would be useful to compare these results with the technique of attaining started flow by mass spillage through wall perforations in a single intake. By counting the dots it is possible to determine how many stages are needed to change the Mach number from any Mach number to any other Mach number. For example, 5 stages will reduce the Mach number from 7 to 4, so that approximately $5/3 \times \Delta M$ stages are required to change the Mach number by ΔM. Scramjet intakes are required to reduce the Mach number by 2/3, i.e. $M_3 = M_1/3$. The red symbols indicate how many stages are required to do this at any given freestream Mach number, M_1. For example, 7 stages are required to bring the Mach number from 6.3 to 2.1. The red curve shows what Mach number reduction is obtained by any stage. For example, the stage at Mach 6.3 reduces the Mach number to 5.7. These results show that quite a few startable stages are required to accomplish a substantial reduction in Mach number using Busemann intake stages when it is required of each stage to start spontaneously.



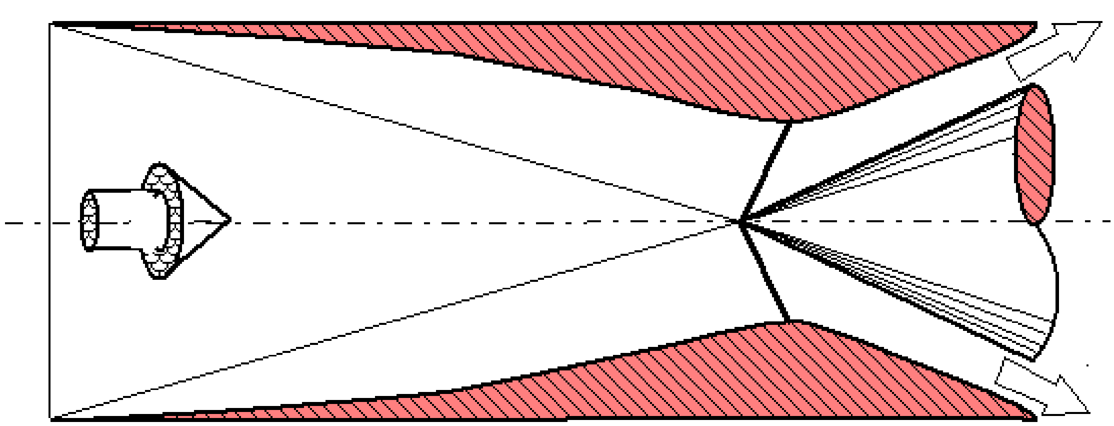
2.6.3 A Busemann/Cone Flow

This type of flow is established by placing the vertex of a solid cone at the vertex of the Busemann shock with its axis aligned with the axis of the Busemann flow,(see poster figure below). The full Busemann/cone flow produces an annular exit flow with a somewhat higher amount of compression at the price of a slight loss of efficiency. Flow over the cone surface is a suitable combustor isolator or it could become the combustor itself if the shock is made to be an oblique detonation wave. Many interesting flow-path variations are possible, suggesting further analysis especially in applications to multi-cycle engines. Modular versions of the Busemann/cone intake produce lift since the exit flow carries some net off-axis momentum. A code has been constructed to calculate the intake performance of the Busemann/cone flow as well as its lift, drag and pitching moment coefficients for a given set of input parameters. It is the first known instance where such an exact calculation provides both intake and lifting surface performance parameters. A parametric study, in conjunction with vehicle performance and trajectory data, would be appropriate.

The Busemann/cone diffuser

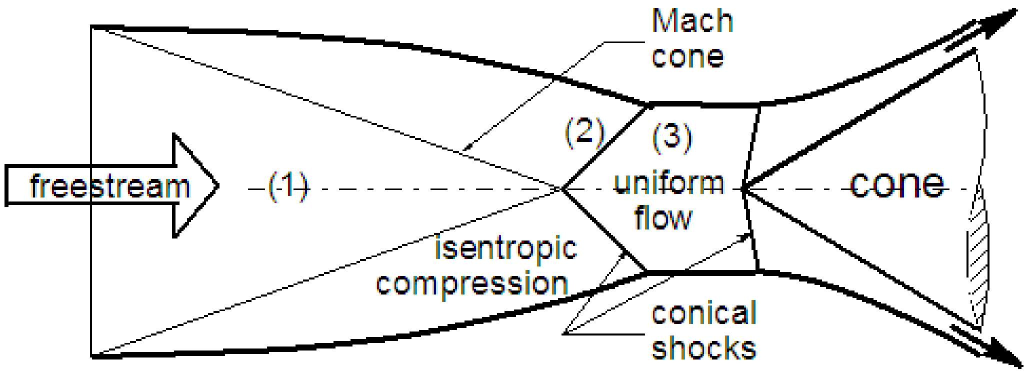
This inlet is made up of three distinct flow types. All flows possess conical symmetry and thus obey the Taylor-McColl equation for conical flow and the oblique shock relations. In the front portion of the inlet, the freestream flow passes through an axially and conically symmetric isentropic compression up to the conical shock. This is the Busemann flow. Aft of the shock the flow is furter compressed isentropically over a cone. It then exits the inlet through an annular passage. At the annulus, the flow has a component of momentum normal to the axis. If an azimuthal modular segment of this flow is selected then this segment will generate lift and a pitching moment (and drag of course). It is interesting to note that, for this inlet, an exact solution of the Taylor-McColl equation will generate flow paths with predictable inlet as well as lifting surface performance.

S.Molder, 1983



2.6.4 Busemann-Duct-Cone Intake

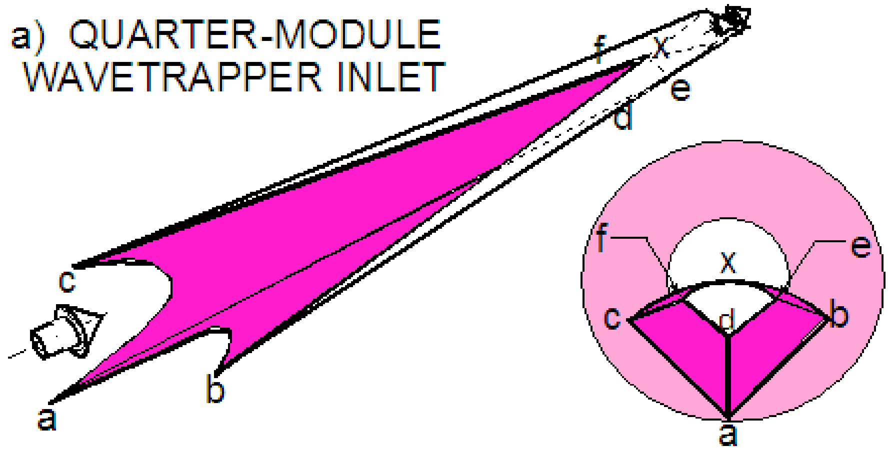
The flow layout is shown in the figure below. Flow between the Busemann and cone shocks is separated by a circular duct (straight pipe). It is a variation of the Busemann-cone flow discussed in Section 2.6.3, above.



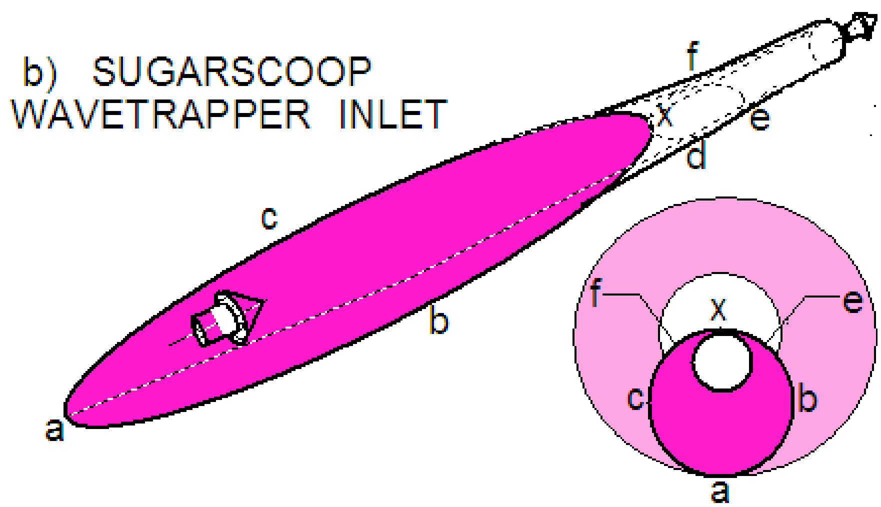
Having two shocks makes it possible to share the total pressure loss between them. This gives a higher efficiency than the basic Busemann and the Busemann-cone combination intakes for the same overall contraction ratio. Flow at the trailing edge of the cone is unaffected by duct length. The duct (isolator) length can therefore be made to suit other requirements. A computer code calculates optimized versions of this intake, the optimization being based on equal pressure ratios across the two shocks. Considerations about lift and drag are similar to those for the Busemann-cone intake. A parametric study should be conducted of the performance of these types of intakes. The basic Busemann flow has been shown to be useful as a starting point for conveniently and rationally designing intakes for hypersonic engines. Two new Taylor-MacColl-based intake geometries are presented for application to annular flow-path combustors. In this chapter we presented and discussed the class of intakes based on the Busemann flow. It seems to be highly suitable for designing air intakes for high Mach number air-breathing engines.

2.6.5 Wavecatcher (Waverider) Intakes

The symmetric, easily understandable and calculable flows can be used not only with their original defining symmetric walls and shocks but also with a wide variety of seemingly unsymmetric wall shapes by embracing two self-evident principles of inviscid supersonic flow behind a shock. The first principle states that the velocity component normal to the streamline is zero. The second principle states that adjacent streamlines form an impenetrable streamline sheet and that, in steady inviscid flow, any such sheet can be replaced by a fixed wall surface. If the streamline sheet originates from a closed, non-intersecting curve on a shockwave surface then the resulting stream-tube contains all the mass flow that has entered the circumscribed portion of the shock and the concomitant stream-tube can be replaced by a fixed duct that captures the shock at its entrance. The principle is similar to that used for the design of wing surfaces where the method is called *wave riding* and is used to obtain pressure distributions on lifting surface shapes for wings with attached leading edge shocks. In application to flow-path design for intakes, where the technique guarantees an attached leading edge shock and no flow spillage, it is more appropriate to call it *wave capturing or wave catching.* Allowable arbitrariness of the curve traced on the shock can produce a stream-tube that looks unsymmetric and three-dimensional despite the fact that it contains highly symmetrical and easily calculable flow. Of course the flow symmetry is preserved only at the entry design Mach number of the basic, original symmetric flow. At the same time, as the flow is simple to calculate, it allows the designer to produce an infinite variety of flowpath shapes to generate the intake duct shape. Such ducts are called *modules* and an intake consisting of an assembly of ducts is called a *modular intake.*



abxc is the freestream capture tube
dexf is the exit stream-tube



circle abxc is the freestream capture tube
smallest circle is the exit stream-tube

Waverider and wavecatcher shapes derive their usefulness from the patching of known (usually simple) flowfields to produce some desired aerodynamic performance. The method consists of projecting a suitable freestream streamtube onto the leading edge wave of a known supersonic flow and then tracing the streamline sheet (in case of wings) or the streamtube (in case of intakes) downstream from the curve formed by the intersection of the freestream tube and the leading edge wave. The resulting surface is then the desired shape and it is readily determinable; the internal flow is unchanged from the original, simple flowfield. The approach is design-oriented in that the flowfield is specified and the streamtube/wall surfaces are calculated. The waverider idea has been applied to the generation of wings and wing/body combinations. We apply the same principle to the generation of intake surfaces and call it 'wavecatching' because all leading edge waves are ingested into the streamtube/intake.[5] Any compressive supersonic/hypersonic flow that has a uniform stream as its upstream boundary is suitable for use as a basic flow for generating the intake flow streamtube. The designer's choice of freestream Mach number and intake contraction or compression is limitless. So is the shape of the freestream capture streamtube. A 90 deg quadrant has been used in the top sketch, above, to generate a module of Busemann flow that can be assembled into a unit with three other modules to give a four-module intake with a circular cross-section as in the designs shown in Section 1 and 2, above. A circular streamtube capture cross-section produces a circular exit flow that leads into a circular cross-section combustor. Jacobsen et al (2006) tested a Busemann type intake, designed for Mach 7, with a sliding throat hatch at a test Mach 4. Partial starting was observed. The intake may have been overcontracted. Such intakes derive their main usefulness from being able to spill mass flow overboard during the flow starting process. In generating the wavecatcher versions, the freestream tube is passed through the focal point of the Busemann flow. This has the effect of generating intake surfaces with the highest possible amount of leading edge sweep and the greatest amount of overboard spillage potential.

The pie-shaped intake is sometimes called 'modular' to highlight the possibility of mounting four such intakes back-to-back to produce a four-module intake with a circular outer shape and four propulsive streams. The key for producing a large open area for overboard spillage is to have the freestream tube surface intersect the leading edge Mach wave of the Busemann flow at its downstream end (apex). This will lead to highly swept leading edges. Wavecatching has been applied to the generation of intake shapes by Keirsey (1965), Molder and Romeskie (1968), Smart (1999) and Matthews (2003). Jacobsen et al (2006), Smart (1999) and Matthews (2003) used the flow inside an axisymmetric conical duct as a basis. This axisymmetric flow can be calculated by the Method of Characteristics. The leading edge wave is curved so that the flow at the exit is rotational. There is an unavoidable Mach disk at the centre line [Rylow, 1990] which is excluded from the freestream tube capture because it cannot be calculated by the Method of Characteristics. This leads to a small degree of inexactness in the resulting intake surface. Using the Method of Characteristics, Matthews also calculated the flow inside an axisymmetric duct whose surface streamline was set at a constant pressure. Smart (1999), has conducted extensive development and testing of a side-spilling intake with an intake flowpath shape transition from rectangular to elliptic.

We will examine the startability of wavecatcher intakes based on Busemann flow because the started flow in Busemann intakes, as well as in its wavecatcher derivatives, is well understood and easily calculable. Also, the wavecatcher stands a chance of **starting spontaneously without the aid of diaphragms, perforations or variable geometry.**

3.0 STARTING OF HIGH-PERFORMANCE INTAKES

Shock swallowing and intake flow starting is an inherently unsteady flow process. If slow enough, i.e. if the shock speed is much less than the surrounding flow speed, then the flow is *quasi-steady* and steady-flow equations are applicable. If the intake is designed 'just to start' then the shock motion is naturally slow. A rapidly moving shock is a sign of an over-started intake that is not contracted as much as possible and some performance loss can be expected. Application of the steady-flow equations leads to the Kantrowitz[6] criterion

[5] No external sonic boom is produced.

[6] The EKD criterion stands for Eggink, Kantrowitz and Donaldson. It is also known as the Kantrowitz/Donaldson criterion and the Kantrowitz criterion. .

for supersonic flow starting, by shock swallowing, in a convergent impermeable duct. The criterion states that the shock, in front of the duct, will be ingested and the flow will become supersonic throughout if the exit of the duct is unchoked and that this occurs when the ratio of exit-to-entry area of the duct is greater than the **Kantrowitz criterion:**

$$\left(\frac{A^{**}}{A_1}\right) = \frac{1}{M_1}\left[\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Above this curve an intake will start spontaneously however, the attained compression is insufficient. A spontaneously started intake, above the Kantrowitz line, will have to be further contracted, to operate near the isentrope, so as to obtain useful performance. Experiments on wind tunnel diffusers and engine intakes have shown that, for intake starting, the Kantrowitz criterion is overly pessimistic and intakes can, in fact, be started at a higher contraction [VanWie and Molder (1992)]. Obviously it becomes more and more difficult to start an intake the closer one gets to the isentrope.

The **isentropic limit** is given by,

$$\left(\frac{A^{*}}{A_1}\right) = M_1\left[\frac{(\gamma+1)}{2}\right]^{-\frac{\gamma+1}{2(\gamma-1)}}\left[1+\frac{(\gamma-1)}{2}M_1^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}}$$

which is the adiabatic, no-loss, contraction line, representing the theoretically highest attainable compression at a total pressure recovery of 100%. Steady intake operation below, or even too close to, this line is impossible. Both curves are shown in the two figures below. A duct with its exit-to-entry area ratio, A_e/A_1, lying between these two limits will operate stably and supersonically if it has been started. Successful starting will depend on what starting technique is used and just where the value of A_e/A_1 lies in the range between A^*/A_1 and A^{**}/A_1. A value of A_e/A_1 close to A^*/A_1 means that the intake is difficult to start whereas a value close to A^{**}/A_1 means that little has to be done to get the intake started.

3.1 The Startability Index

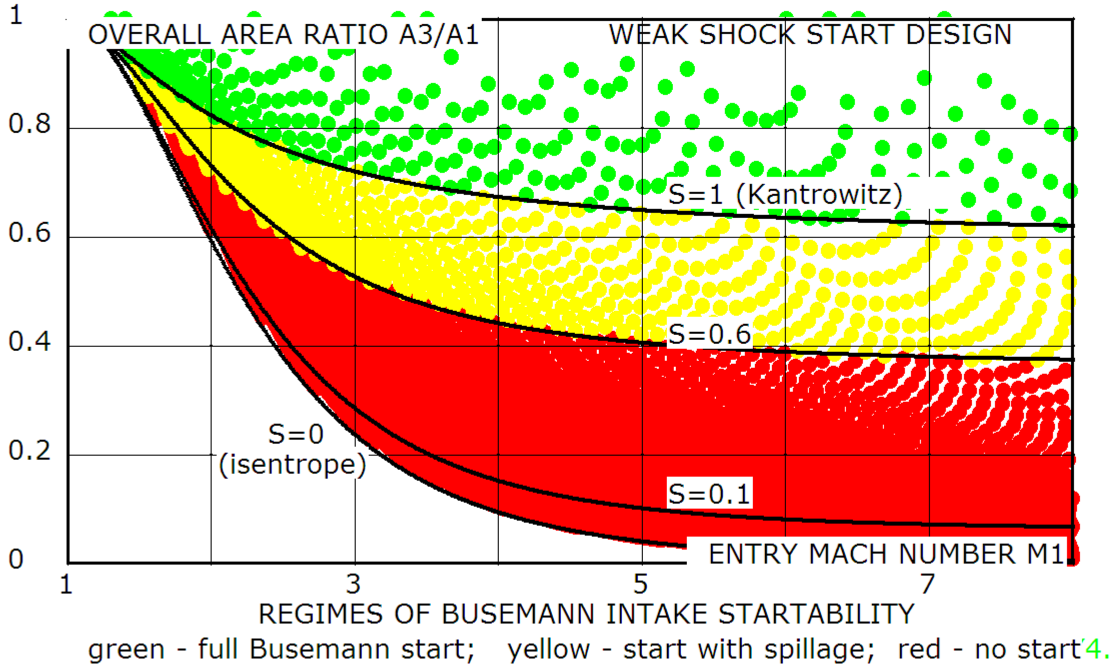
We propose an index, S_i, to measure the difficulty of starting an intake. It is a measure of the position of the intake's contraction ratio on the scale between the Kantrowitz and the isentropic contraction ratios,

$$\boxed{S_i = \frac{A_e/A^* - 1}{A^{**}/A^* - 1}}$$

This index is a ratio of contractions and, with the presumption that startability increases with a decrease in contraction, we call it the **startability index** and hope that it can be shown to be a meaningful measure of startability. At the Kantrowitz condition, where the intake just starts spontaneously, the startability index has a value of 1 and at a contraction which corresponds to the isentropic area ratio, below which the intake will not remain started, the startability index takes on a value of zero. The index can be applied to the entire intake as well as to the internal contraction portion of the intake to reflect their individual propensities to start spontaneously.

3.2 Enclosed Ducts

A series of Busemann intake calculations were done for the axisymmetric Busemann intake through a range of M_2 from 1 to 8 with k varying between 1 and M_2. The results were plotted on the graph below for overall area ratio A_3/A_1 against the freestream Mach number M_1. A green point was plotted whenever the Kantrowitz criterion indicated that the full intake would start spontaneously.



Full Busemann intakes will start and operate in the region above $S = 1$. In this region, as noted by Stockbridge (1978), there is not enough contraction to derive useful performance from a **full Busemann intake**. The freestream Mach number is reduced by, at most, half a unit, e.g. from 8 to 7.5.

3.3 Steady-State Starting of Modular Half-Busemann Intake – Weak Shock

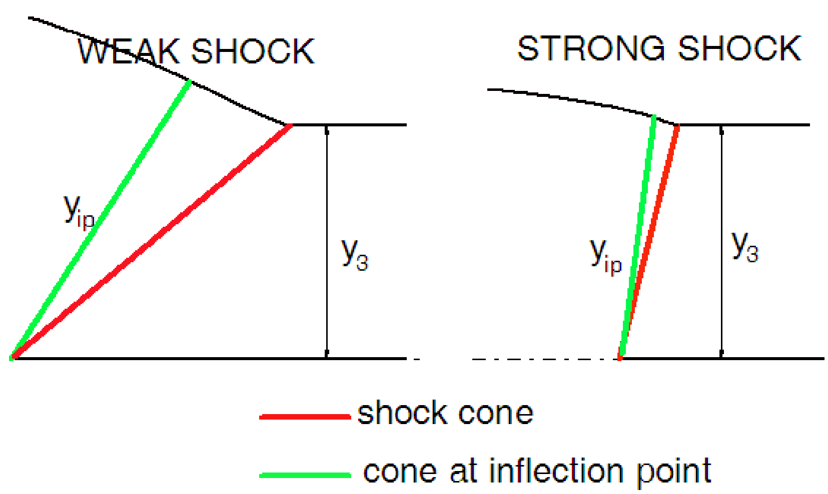
Modular intakes are characterized by leading edges that are swept back to the focal point of Busemann flow. Such intakes are shown in the figures in Section 2.6.5. The basic flow for the weak shock is obtained by selecting a value of k just above 1. For a half-Busemann intake (see Sect 4.1), a semi-circular freestream tube is projected onto the leading wave of the Busemann flow and streamlines are traced from the intersection of the streamtube and the wave into the downstream flow. During starting a more-or-less normal shock will move into the V-shaped gore. Flow will be spilled through the gore as long as the shock is upstream of the V-notch. On reaching the V-notch the shock will become conical at the inflection point. The flow at the inflection is convergent and everywhere normal to the shock. The Mach number on the upstream side of the shock is known so that this Mach number, the cone area and the exit area can be used to calculate the startability of the contraction from the inflection point to the exit. In this situation the flow up to the inflection point is already started and the shock will be swallowed if the startability index of the internal contraction is greater than one. The use of spillage through the gore has allowed starting at a much lower overall startability index. The intakes in the yellow area will start. Busemann intakes, based on the weak-shock design will not start in the red area. The best Mach number reduction in the yellow area is about one unit, i.e. from 8 to 7.

This is still not enough for adequate intake performance. It is interesting to note that the lower boundary of the weak shock startability limit for weak shocks coincided almost exactly with a constant overall startability index of 0.6. There is no clear reason why this should be so but it does illustrate the usefulness of the startability index in that it becomes a simple matter to determine if a weak-shock Busemann intake can be started, simply from knowing its overall contraction ratio and freestream Mach number.

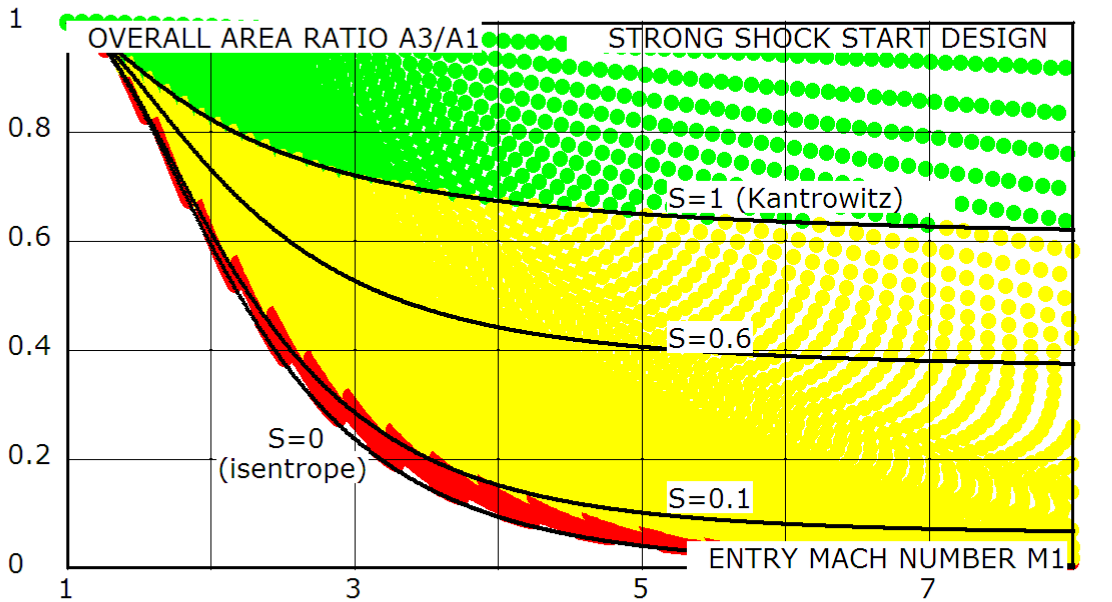
3.4 Steady-State Starting of Modular Half-Busemann Intakes – Strong Shock

Note that the 'shock polar' Eqn. 2.11, gives two solutions for the shock angle, a weak and a strong shock, for given M_2 and δ_{23}. These would produce two different Busemann intake contours; the weak shock version would have supersonic and the strong shock would have a subsonic exit flow. It turns out that the weak shock version is better suited for scramjet application; however, it has too much internal contraction to be self-starting. The strong shock version has inferior performance but has less internal contraction and will self-start. These thoughts open a possibility – design a wavecatcher Busemann intake, with a strong shock, such that it starts spontaneously; then reduce the back-pressure to draw the strong shock downstream and obtain a supersonic exit flow compatible with scramjet operation. In doing this we note that the flow, up to the location of the strong shock, has not changed so that the intake remains on design and started, with no spillage. In fact, the amount of internal contraction remains the same and we could really start the inlet with the weak shock structure in the first place. So that the strong Busemann shape is really a design tool which leads to a modified Busemann flow but with a started inlet of high external compression having a supersonic exit Mach number. A drawback is that the exit flow is no longer conical and a lower efficiency is established, although its axial symmetry is preserved.

A strong shock Busemann flow contour is obtained by selecting M_2 and k values such that k is just slightly smaller than M_2. This produces a near-normal strong shock at the focal point of the Busemann flow with the V of the gore longitudinally situated close to the foot of the strong shock. A sketch comparing the geometry of the weak and strong shock cases is shown below. In each case the conical Busemann shock is shown in red, spanning the origin and the corner while the conical normal shock is in green spanning the origin and the inflection point. For each case the amount of internal contraction, that is critical to starting, is represented by the area ratio that is directly proportional to $(y_3/y_{ip})^2$. This area ratio is smaller for the weak shock than for the strong shock case. The strong shock case, with less internal contraction, is therefore easier to start.



The strong shock initial conditions were used to start a large number of Busemann intake calculations throughout the range of M_2 and k. In each case the intake was examined to see if it would start as a complete Busemann intake (without spillage). If it did, a green point would be plotted in the area ratio vs. Mach number plot below. If its **internal contraction** would start (necessitating spillage) then a yellow point would be plotted. No start, in either case, would result in a red point. From this plot it appears that the startability of Busemann intakes has been extended considerably by using the strong shock as a boundary condition for generating the basic flow in the intake. It is an attempt to answer Stockbridge's (1978) criticism of the startability of Busemann intakes.



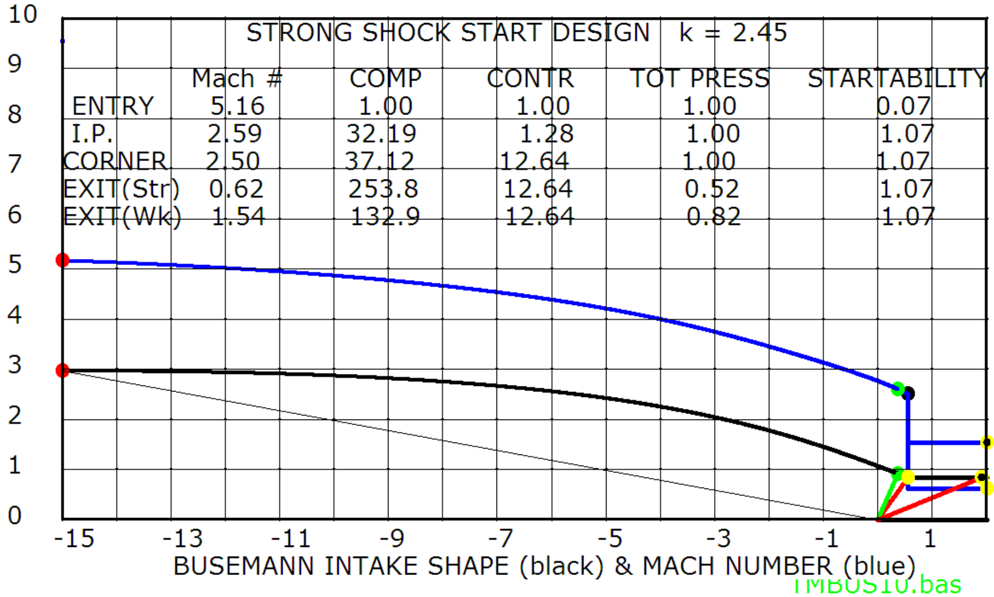
REGIMES OF BUSEMANN INTAKE STARTABILITY
green - full Busemann start; yellow - start with spillage; red - no start

The lower bound of the starting region is at a startability index of S ≈ 0.1 so that quite useful high-contraction intakes are indicated to start. If the strong-shock back pressure is maintained the strong shock will take up its design position and the intake will operate with a subsonic exit Mach number. However, the efficiency will be quite low because of the strong shock. (Total pressure recovery will be that of a normal shock at Mach k).

A lowering of the back-pressure will cause the strong shock to move downstream and a weak shock will establish itself at the focus. The weak shock will yield a supersonic exit flow and a high efficiency, suitable for scramjet application. The flow deflection through the weak shock is the same as through the strong shock. An expansion fan will occur at the corner that will interact with the weak shock to produce a non-uniform exit flow. This is the major drawback of this type of intake design for starting. A detailed sample calculation is described in the next section.

3.5 An Example of Startable High-Performance Intake Design from Strong Shock

The black curve in the figure below is the Busemann intake contour. The curved surface extends from the red dot, freestream conditions, and stops at the corner and is continued downstream from there by a straight 'isolator' wall. There is an inflection point in the wall shape at the green dot. A 'cowl' extends downstream from the origin (0,0). The upstream red line, extending from the origin to the corner is a possible position of a strong shock. The downstream red line is a possible position of a weak shock. The blue curve shows a gradually decreasing surface Mach number from 5.15 in the freestream to 2.5 just before the strong shock.



As shown in the table, the Mach number then drops to 0.62 for the strong shock or 1.54 for the weak shock, in the isolator. This is shown by the two blue horizontal lines. The design is based on a pre-shock Mach number of 2.5 and a normal Mach number, k = 2.45. The second column gives the pressures in the various locations as multiples of the freestream pressure. A strong shock, at the isolator entrance will produce an isolator pressure of 253.8, whereas a weak shock will produce a pressure of 132.9 times local atmospheric. The contraction ratio from entry to exit is 12.64. The contraction ratio from the inflection point to the exit is 1.28. This corresponds to an area ratio of 0.78. An enclosed duct with this amount of internal contraction will swallow a normal shock at Mach 2.5 according to the Kantrowitz criterion. Its startability index is 1.07. The full Busemann intake is far from starting with a startability index of 0.07. The starting task then consists of spilling enough mass flow overboard to entice a normal (conical) shock to take up its position at the inflection point. Lowering the back-pressure to 253.8 would cause the normal shock to become a strong shock at the corner, with uniform subsonic exit flow at Mach 0.62 and total pressure recovery of 0.52. A further lowering of pressure to 132.9 will cause a weak shock to appear at the cowl with Mach 1.54 at the exit. An expansion fan appears at the corner, which interacts with the shock, causing a non-uniform exit flow. The total pressure recovery is 0.82 and the intake is started.

This is a first cut at attempting a startable design. It gives an idea of the intake's capability from the entry and exit Mach numbers, an idea of the efficiency from the total pressure recovery and an indication that the intake can be started by mass flow spillage from the startability index.

4.0 THREE STARTING INTAKES

In this section we present examples of flow starting in three different intakes:

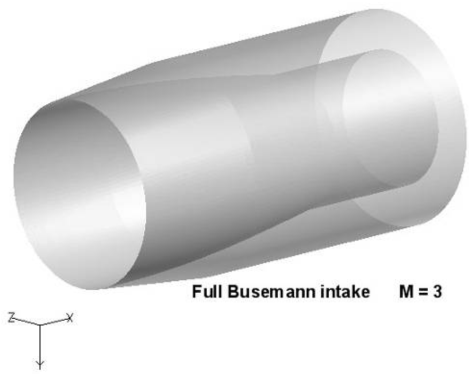
a) Two half-Busemann intakes – a CFD simulation

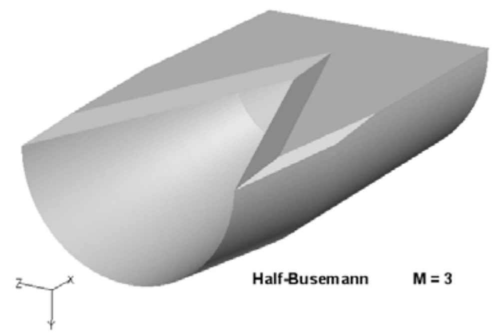
b) A quarter-Busemann modulated intake – experiments in guntunnel

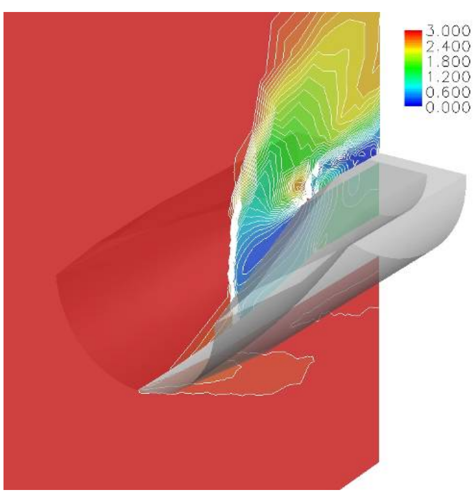
c) A seven-module intake – experiments in guntunnel

4.1 The Half-Busemann Intake – Starting by Overboard Spillage

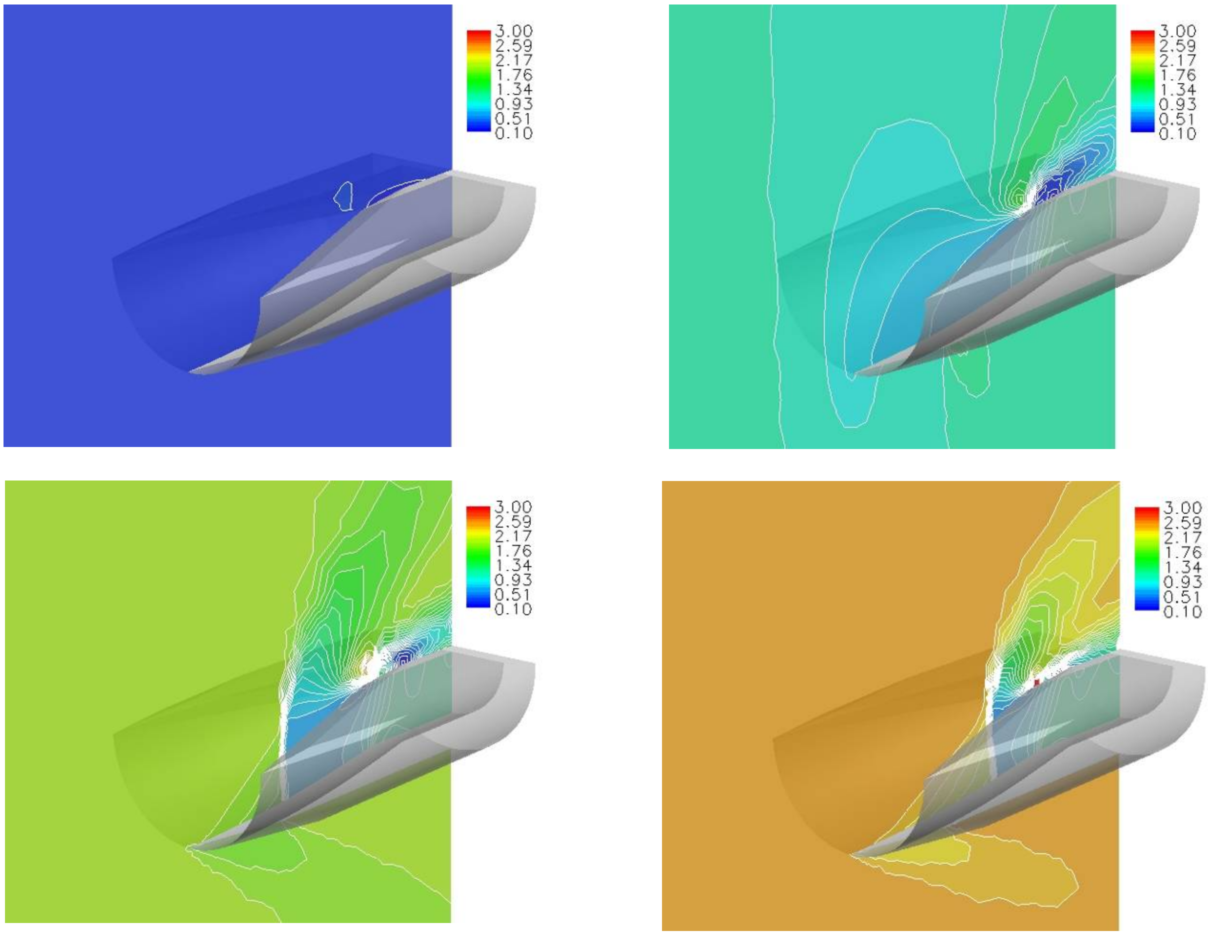
A full-Busemann and a half-Busemann intake are shown below. The full Busemann is the basic flow for the half-Busemann. The starting flow in the half-Busemann has to end up being the same as in the full Busemann intake when started. CFD calculations were done, using the time-realistic Solver III, for starting two half-Busemann intakes, for area ratios of 0.398 and 0.510. These two area ratio intakes were predicted not to start and to start, respectively, for a weak shock started mode. The purpose was to demonstrate overboard mass spillage as a mechanism for intake starting.

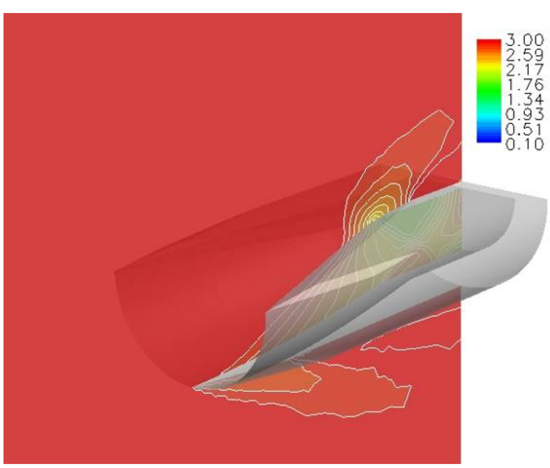






The CFD result is shown on the left for the higher contraction (0.398) intake at a freestream mach number of 3.00. Mach numbers are depicted in the colours. The centre line plane is shown in the flow-field. A strong shock has stabilized in the gore with subsonic downstream flow. The intake remains unstarted. Similar CFD calculations were done for the less contracted (0.510) half-Busemann for a selection of Mach numbers from 0.5 to 3.0. Results are shown in the five diagrams below. For this contraction ratio, started flow was attained by the weak shock mechanism at Mach 3.0 (bottom figure with red background).

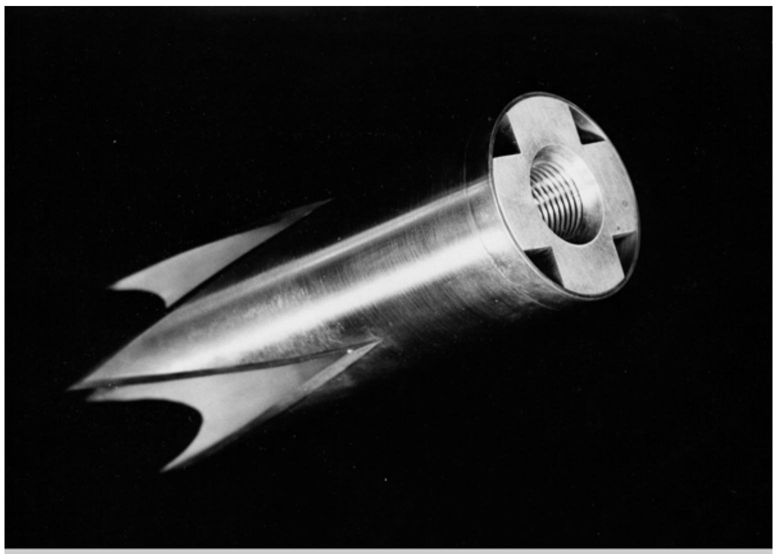




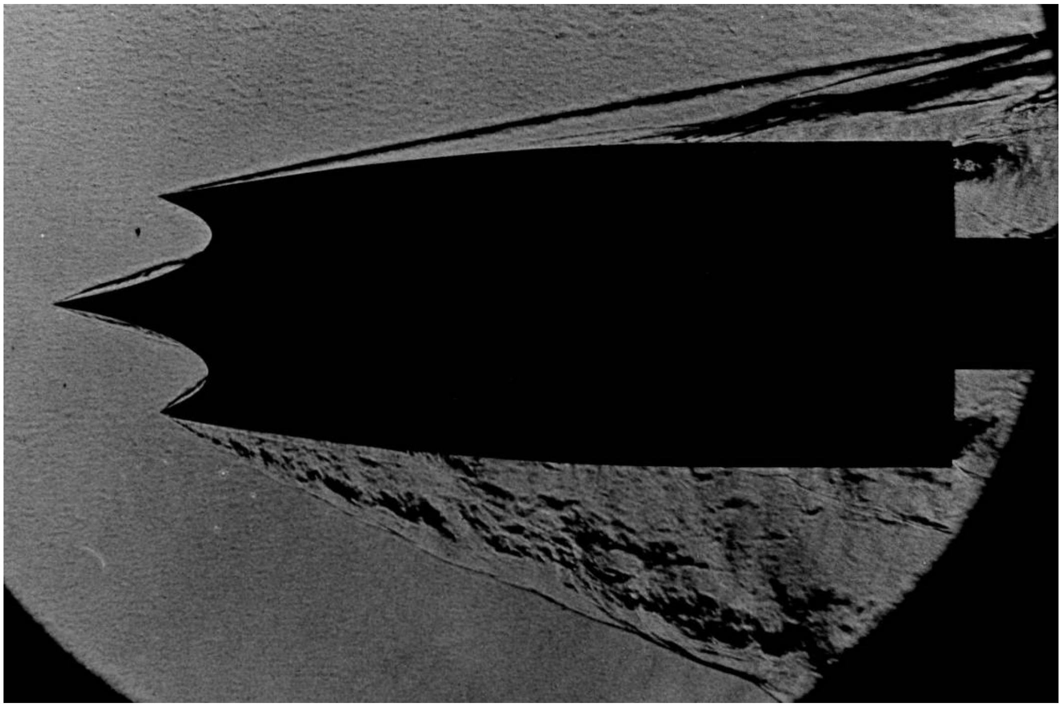
At Mach 3, a Busemann intake with an area ratio below 0.526 should not start by overboard spillage. The internal contraction is too high. However, in the figure on the left, the half-Busemann intake is started at an area ratio of 0.510. Results of started flow, both computational as well as experimental, below the Kantrowitz limit for starting, have been observed previously [Molder (1992), VanWie (2000)]. The 1D limit seems to be too pessimistic when applied to multidimensional flow.

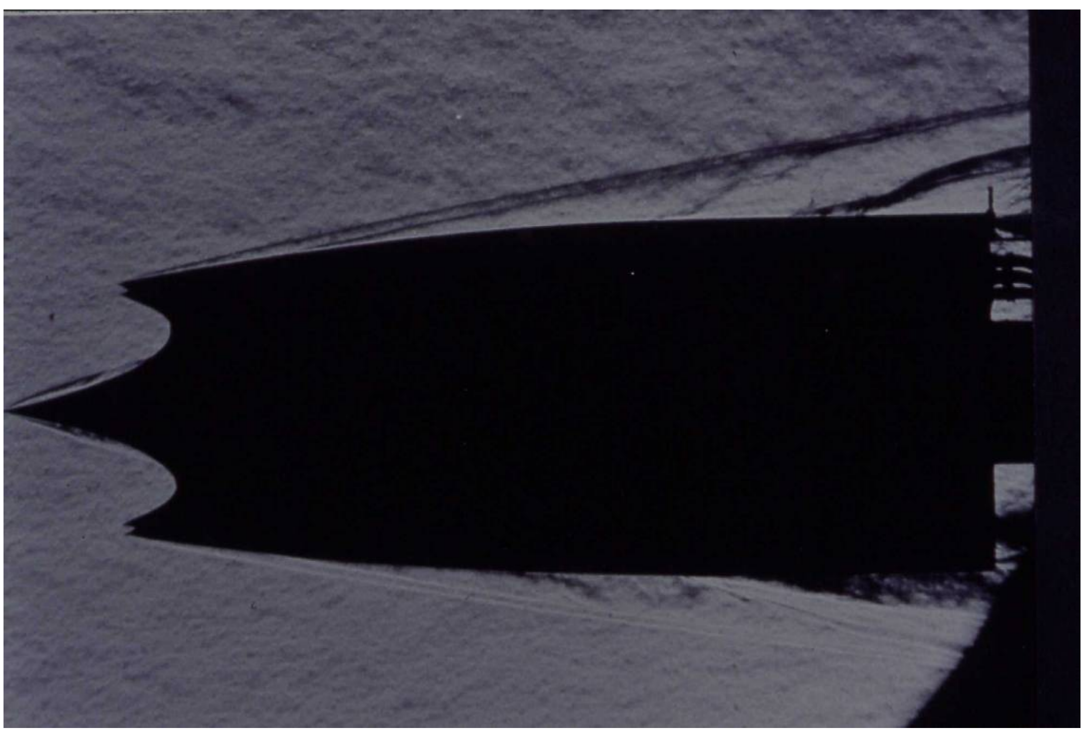
4.2 Quarter-Busemann Modular Intake; Impulse and Spillage Start

A four-module intake, pictured below, was designed, based on wavecatching Busemann flow, as described in Section 2.6.5. The Busemann design is for a freestream Mach number of 8.33 and an area ratio of 0.08. Tests were conducted in a piston-driven guntunnel [Molder and Romeskie, 1968] at a nominal total pressures and total temperatures of 1000 atm and 1000 K and a test-section Mach number of 8.33. A schlieren picture is shown below where the top module is started and the bottom module is unstarted. The next picture shows all modules with started flow. In these tests starting was greatly enhanced by an initial high vacuum in the test-section and, as well, by the impulsive nature of the starting flow from the gun-tunnel nozzle.



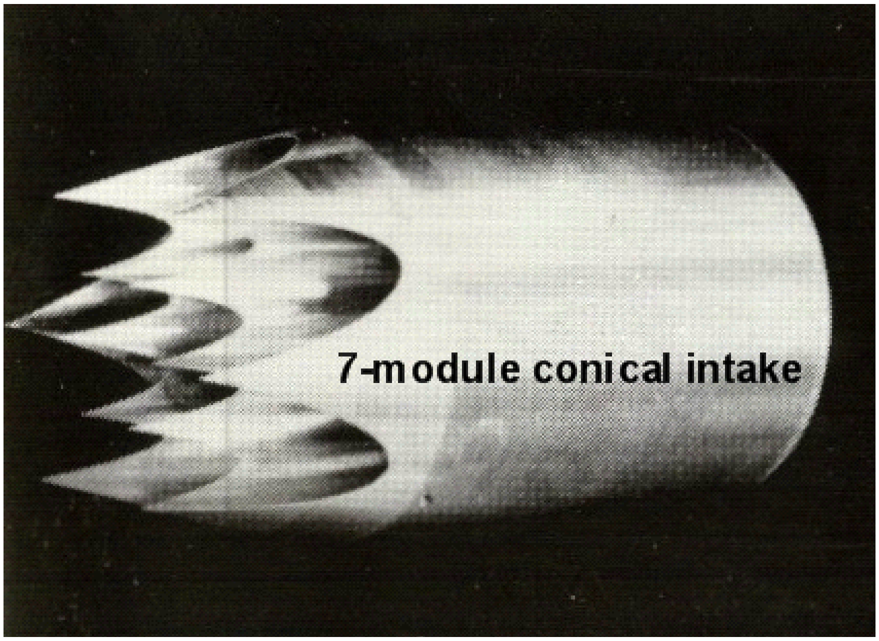
quarter modulated Busemann intake

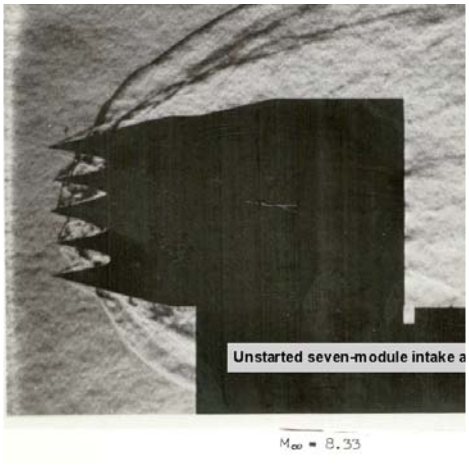


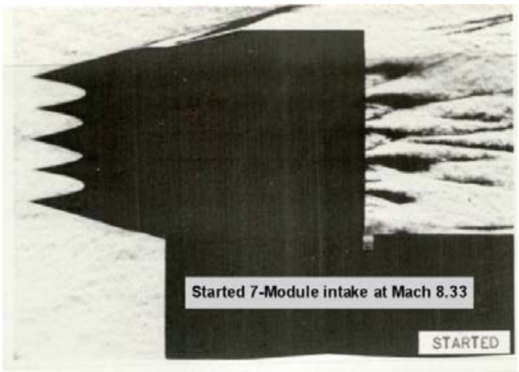


4.3 Seven-Module Intake; Start by Impulsive Flow

A seven-module intake was constructed for gun-tunnel tests at Mach 8.33. The purpose of these tests was to show that starting of high contraction intakes can take place entirely impulsively since not much spillage can be expected, especially from the central module. The intake consists of one central module surrounded by six identical modules as shown in the picture below. Each module is axisymmetric and conical with a 10 degree included cone angle. Such intakes, although difficult to start, provide very short flow paths with minimal viscous losses. Down-scaling to grid configurations can be envisaged.







The left picture shows unstarted flow in the multi-module intake with a contraction ratio of 13:1. A contraction ratio of 10:1 led to impulsively started flow as shown in the right hand figure.

5.0 CONCLUDING REMARKS

Thermodynamics of Busemann flow make it suitable for use as basic flow to obtain high-performance intake flows for scramjet engines. Application of streamline tracing (wavecatching) to Busemann flow yields flow-path geometries suitable for scramjet intakes.

Startability and contraction of intakes is defined and a **startability index** has been introduced as a measure of startability and contraction. For Busemann intakes, it has a value of 0.6 for intakes that start by overboard spillage from a weak shock starting design and a value below 0.1 for strong shock starting designs.

A design approach has been presented that starts by specifying the exit conditions of a Busemann type intake such that it gives an idea of the capability from the entry and exit Mach numbers, an idea of the efficiency from the total pressure recovery and an indication that the intake can be started by mass flow spillage from the startability index.

Several fixed-geometry modular intake designs are reviewed.

There remain some additional issues, within the design cycle, that have to be critically examined:

1) Effect on performance resulting from flow non-uniformity due to the strong-shock starting technique;
2) Attainment of sufficient spillage through holes, slots, slats or gores to get the normal shock sufficiently far into the intake to be swallowed in the internal contraction;
3) Effects of viscous flow.

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